$L(2,1)$ Labeling For Regular Graphs

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***Abstract*—A $k-L(2,1)$ labeling *f* for a given graph $G=(V,E)$, is a function $f:V\to\{0,1,2,...,k\}$ such that for every pair of vertices *x*, *y* in V, $|f(x)-f(y)|\geq 2$ if $d(x,y)=1$, and $|f(x)-f(y)|\geq 1$ if $d(x,y)=2$ where $d(x,y)$ denotes the distance between vertices *x* and *y*. The $L(2,1)$ labeling problem is finding the minimum *k* such that *G* has a $k-L(2,1)$ labeling. This paper established the bounds of $L(2,1)$ labeling, for some regular graphs such as star-like, pancake, burnt pancake, and folded hypercube graph.**

***Index Terms*— $L(2,1)$ labeling, star-like, pancake, burnt pancake, folded hypercube.**

I. Introduction

Due to fast growth in the use of radio frequencies and the scarce of radio frequencies, allocate these finite frequencies efficiently is very important. If two radio stations are closed to each other in the same area, they must not be assigned too closed frequencies otherwise it will cause some interferences. The problem of efficiently allocating finite frequencies to avoid interference, which is called the channel assignment problem, becomes very important. The channel assignment problem finds the minimum range of frequencies for all transmitters. The frequency assignment problem which was modeled by graph was originally introduced by Hale [8] in 1980. In order to avoid disturbing each other, they use the vertices to denote the transmitters and the edges to indicate two transmitters being “very closed”. When two transmitters are adjacent which means they are “very closed”, they should use frequencies differ by at least *p*; when two transmitters are at distance two, which means they are “closed”, they should use frequencies differ by at least *q* where $p>q$. In general, a $k-L(p,q)$ labeling f for a given graph $G=(V,E)$ with positive integers p and q where $p>q$, is a function $f:V\to\{0,1,2,\cdots,k\}$ such that $|f(x)-f(y)|\geq p$ if $d(x,y)=1$, and $|f(x)-f(y)|\geq q$ if $d(x,y)=2$ where $d(x,y)$ is the distance between vertices x and *y*. The $L(p,q)$-*labeling number* $\lambda_{p,q}(G)$ *of G* is the minimum *k* such that there exists a $k-L(p,q)$ labeling of graph *G*. *The* $L(p,q)$-*labeling problem* is the problem of finding the $L(p,q)$-*labeling* number of graphs which has been proved to be NP-Complete [7].

For special numbers of *p* and *q*, Griggs and Yeh [7] brought up $L(2,1)$-*labeling* in 1992. Some surveys of the results on $L(2,1)$-*labeling* problem are given in [3][4][13].

The *n*-dimensional graphs are very interesting and brought themselves much attention. Griggs and Yeh [7] established the $L(2,1)$-*labeling* number for *n*-dimensional hypercube Q_n to be $\lambda_{2,1}(Q_n)\leq 2n+1$ for $n\geq 5$. Whittlesey [10] improved the upper bound by one, for all *n*. For the lower bound, Jonas [9] has shown that $n+3\leq\lambda_{2,1}(Q_n)$ and $n+4\leq\lambda_{2,1}(Q_n)$ for n =8,16. This paper established the bounds of $L(2,1)$-*labeling* number for some other regular graphs such as star-like graphs, pancake graph, burn pancake graph, and folded hypercube graph. These graphs are popular in the interconnection network topologies. The interconnection network plays a important role in determining the whole performance of a multi-processor system. Hypercube-type networks are developed over the past few years since they propose the rich interconnection structure.

II. STAR-LIKE GRAPH

The n-dimensional star-like graph S_n defined in [1], is a graph in which the vertices are denoted by a sequence number of distinct permutation of integer set $\{1,2,3\cdots,n\}$. Two vertices are adjacent in S_n if they can be obtained by exchanging first digit (the leftmost digit) with i-th digit where $1 < i \le n$. So that S_n is a n-1 regular graph containing $n!$ vertices. Figure 1 shows an example of a 4-star-like graph which contains 24 vertices; vertex 2341 is adjacent with vertices 1342, 3241, and 4321. A star-like graph S_n contains n disjoint S_{n-1} subgraphs $\{x_1x_2\cdots x_{n-1}1, x_1x_2\cdots x_{n-1}2$, $\cdots, x_1x_2\cdots x_{n-1}n\}$ where $x_1, x_2, \cdots, x_{n-1} \in \{1,2,\cdots,n\}$ and $x_i \ne x_n$ in each subgraph for $1 \le i \le n-1$, such that each pair of S_{n-1} is connected by $(n-2)!$ edges.

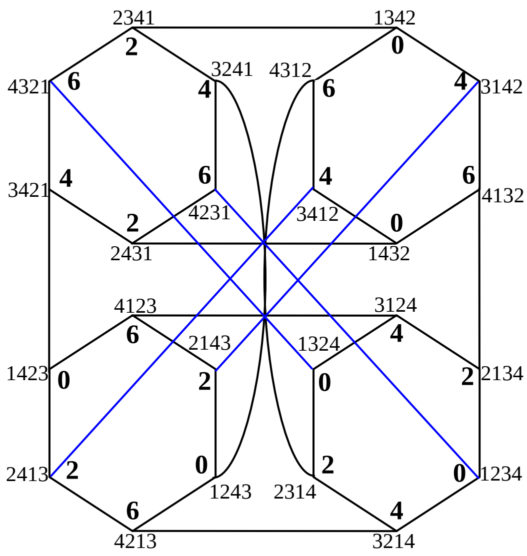

Figure 1. A 6-$L(2,1)$ labeling of S_4.

Since the vertex in a star-like graph is exchanged first digit (the leftmost digit) with i-th digit of its neighbors where $1 < i \le n$, it takes at least three steps to exchange back to the same first digit. Hence we have following proposition.

Proposition 1: *The vertices with the same first digit in an n-dimensional star-like graph S_n are at distance at least 3.*

The following lemmas are used in the proof of our theorems.

Lemma 1. [13] *Let H be a subgraph of graph G. Then* $\lambda_{d_1,d_2}(H) \le \lambda_{d_1,d_2}(G)$ *for* $d_1 \ge d_2$.

Lemma 2. [7] *If graph G has three vertices of maximum degree n such that one such vertex is adjacent to the other two , then* $\lambda_{2,1}(G) \ge n+2$.

Lemma 3. [12] $\lambda_{2,1}(C_n) = 4$ *for* $n \ge 3$.

Since the vertices in complete graph K_n are adjacent to each other, the $L(2,1)$ labels of each pair of vertices has to be differ by at least two. In a complete bipartite graph, each pair of vertices in the same partite set is at distance two. Two vertices in different partite set are adjacent, hence every vertex has to have distinct $L(2,1)$ label. Therefore, we have following proposition.

Proposition 2. $\lambda_{2,1}(K_n) = 2n-2$ *for* $n \ge 2$*; and* $\lambda_{2,1}(K_{m,n}) = m+n$ *for* $m \ge n$.

For an n-dimensional star-like graph S_n, since S_1 is trivial; $S_2 = K_2$, and $S_3 = C_6$, by lemma 3 and proposition 2, we have $\lambda_{2,1}(S_1) = 0$, $\lambda_{2,1}(S_2) = 2$, $\lambda_{2,1}(S_3) = 4$. Figure 2 is a 5-$L(2,1)$*-labeling* of S_4, hence $\lambda_{2,1}(S_4) \le 5$. By lemma 2, $\lambda_{2,1}(S_4) \ge 5$ which implies $\lambda_{2,1}(S_4) = 5$. For the case of $n \ge 5$, we have following theorem.

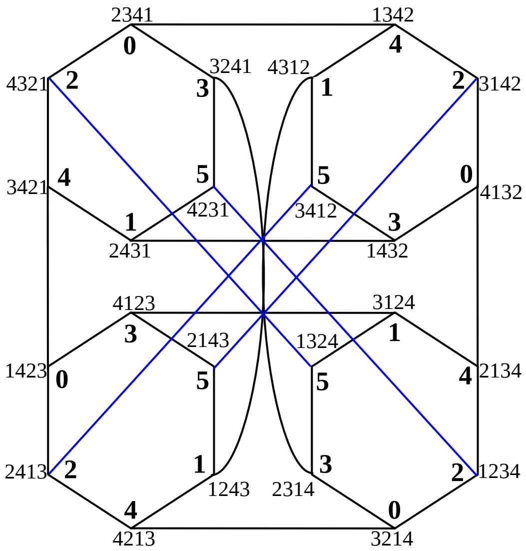

Figure 2. A 5-$L(2,1)$ labeling of S_4.

Theorem 1. *Let S_n be an n-dimensional Star-like graph. Then $n+1 \le \lambda_{2,1}(S_n) \le 2n-2$ for $n \ge 5$.*

Proof: Consider a labeling $f: V(S_n) \rightarrow \{0,2,4,\cdots,2n-2\}$ such that $f(v)=2i-2$ for the vertex v where its sequence number starts with digit i as shown in figure 1. By proposition 1, we know that each vertex with the same $L(2,1)$ label are at distance at least 3. Since only even numbers are used in f, if u and v are adjacent vertices, $|f(u)-f(v)| \ge 2$. Hence f is a (2*n*-2)-$L(2,1)$-labeling of S_n, which implies that $\lambda_{2,1}(S_n) \le 2n-2$. By lemma 2, we have $n+1 \le \lambda_{2,1}(S_n)$. Therefore the proof of theorem completes. ■

III. Pancake Graph

An *n*-dimensional pancake graph PC_n [2] is a graph in which the vertices are denoted by a sequence number of distinct permutation of integer set $\{1,2,3\cdots,n\}$. Two vertices $u=(u_1u_2\cdots u_i\cdots u_n)$ and $v=(v_1v_2\cdots v_i\cdots v_n)$ are adjacent in PC_n if there exists an i, $2 \le i \le n$, such that $v_j = u_{i-j+1}$ for all $1 \le j \le i$ and $v_j = u_j$ for $i < j \le n$. In another words, vertex v represents the *i*-th prefix reversal of vertex *u*, which is then denoted by $(u)^i_{PC}$. For example, $(12345)^4_{PC} = 43215$. So that PC_n is an *n*-1 regular graph containing $n!$ vertices. A 4-pancake graph, shown in figure 3, contains 24 vertices; the vertex 1234 is adjacent with vertices 2134, 3214, and 4321. In general, PC_n contains *n* disjoint PC_{n-1} subgraphs $\{x_1x_2\cdots x_{n-1}1, x_1x_2\cdots x_{n-1}2, \cdots, x_1x_2\cdots x_{n-1}n\}$ where $x_1,x_2,\cdots,x_{n-1} \in \{1,2,\cdots,n\}$ and $x_i \ne x_n$ in each subgraph for $1 \le i \le n-1$, such that each pair of PC_{n-1} is connected by $(n-2)!$ edges.

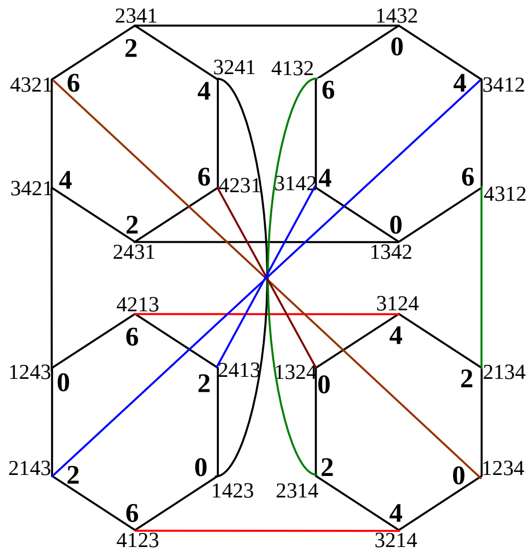

Figure 3. A 6-$L(2,1)$ labeling of PC_4.

Since the vertex in a pancake graph is a prefix reversal of its neighbors, it takes at least three steps to flip back to the same first digit. Hence we have following proposition.

Proposition 3: *The vertices with the same first digit in an n-dimensional pancake graph PC_n are at distance at least 3.*

For an n-dimensional pancake graph PC_n, since PC_1 is trivial; $PC_2 = K_2$, and $PC_3 = C_6$, by lemma 3 and proposition 2, we have $\lambda_{2,1}(PC_1)=0$, $\lambda_{2,1}(PC_2)=2$, and $\lambda_{2,1}(PC_3)=4$. For the case of $n \ge 4$, we have following theorem.

Theorem 2. *Let PC_n be an n-dimensional Pancake graph of graph. Then $n+1 \le \lambda_{2,1}(PC_n) \le 2n-2$ for $n \ge 4$.*

Proof: Consider a labeling $f: V(PC_n) \to \{0, 2, 4, \cdots 2n-2\}$ such that $f(v) = 2i-2$ for vertex v starts with digit i as shown in figure 3. By proposition 3, we know that each vertex with the same $L(2,1)$ label are at distance at least 3. Since only even numbers are used in f, if u and v are adjacent vertices, $|f(u) - f(v)| \ge 2$. Hence f is a $(2n-2)-L(2,1)$ labeling of PC_n, which implies $\lambda_{2,1}(PC_n) \le 2n-2$. By lemma 2, we have $n+1 \le \lambda_{2,1}(PC_n)$. That completes the proof. ■

IV. Burnt Pancake Graph

The n-dimensional burnt pancake graph BP_n defined in [5] is a graph in which the vertices are denoted by a sequence number of distinct permutation of (signed) integer set $\{1(or\ \dot{1}), 2(or\ \dot{2}), 3(or\ \dot{3}), \cdots, n(or\ \dot{n})\}$. Two vertices $u = (u_1 u_2 \cdots u_i \cdots u_n)$ and $v = (v_1 v_2 \cdots v_i \cdots v_n)$ are adjacent in BP_n if there exists an i, $1 \le i \le n$, such that $v_j = \dot{u}_{i-j+1}$ for all $1 \le j \le i$, and $v_j = u_j$ for $i < j \le n$. Therefore, v is a representation of the i-th prefix reversal of signed integers of vertex u, which may be denoted by $(u)^i_{BP}$. For example, $(1\dot{2}3\dot{4}5)^4_{BP} = 4\dot{3}2\dot{1}5$. Hence, BP_n is an n regular graph containing $2^n n!$ vertices. A 3-burnt pancake graph, shown in figure 4, contains 48 vertices; vertex 123 is adjacent with vertices $\dot{1}23$, $\dot{2}\dot{1}3$, and $\dot{3}\dot{2}\dot{1}$. A BP_n contains $2n$ disjoint BP_{n-1} subgraphs $\{x_1 x_2 \cdots x_{n-1} 1, x_1 x_2 \cdots x_{n-1} 2, \cdots, x_1 x_2 \cdots x_{n-1} n, x_1 x_2 \cdots x_{n-1} \dot{1}, x_1 x_2 \cdots x_{n-1} \dot{2}, \cdots, x_1 x_2 \cdots x_{n-1} \dot{n}\}$ where $x_1, x_2, \cdots, x_{n-1} \in \{1(or\ \dot{1}), 2(or\ \dot{2}), 3(or\ \dot{3}), \cdots, n(or\ \dot{n})\}$ and $x_i \ne x_n$ in each subgraph for $1 \le i \le n-1$.

The burnt pancake graph has a similar property as in pancake graph for the same reason, and we stated it as next proposition.

Proposition 4: *The vertices with the same first digit in an n-dimensional burnt pancake graph BP_n are at distance at least 3.*

For an n-dimensional burnt pancake graph BP_n, since $BP_1 = K_2$ and $BP_2 = C_8$, by lemma 3 and proposition 2, we have $\lambda_{2,1}(BP_1) = 2$, $\lambda_{2,1}(BP_2) = 4$. For the case of $n \ge 3$, we have following two theorems.

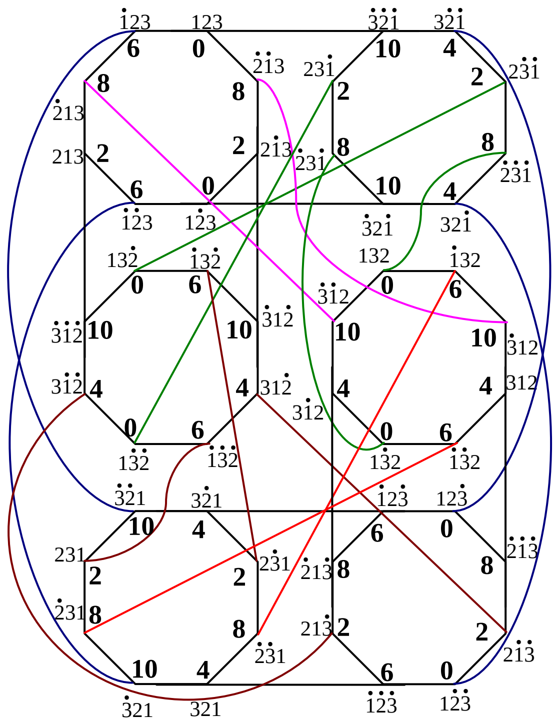

Figure 4. A 10-$L(2,1)$ labeling of BP_3.

Theorem 3. *Let BP_n be an n-dimensional Burnt Pancake graph. Then $\lambda_{2,1}(BP_3) = 6$.*

Proof: We divide the vertices of BP_3 into groups such that the vertices in the same group have the same $L(2,1)$ labels, which implies any two vertices in same group are at distance at least 3. Notice that for each subgraph C_8, there are no more than two vertices in the same group, which implies there are at most twelve vertices in one group of the BP_3. Since BP_3 is 3-regular, there are at least four groups. Suppose there are exactly four groups in BP_3, then each group must include exactly twelve vertices. By the 6- $L(2,1)$ labeling of BP_3 shown in figure 5 and figure 6, we have $\lambda_{2,1}(BP_3) \le 6$. Consider any subgraph C_8 of BP_3, let abc be a vertex in group1, the only vertices in the same subgraph C_8 that can be also in group1 are $\{a\dot{b}c, \dot{a}\dot{b}c, bac\}$. Assume abc and $a\dot{b}c$ are both in group1. Consider the subgraph C_8' end with $\dot{a}$, the only two vertices that can be in group1 are adjacent in C_8' which produces a contradiction. Hence the only two possible cases to divide BP_3 into four groups are shown in figure 5 and figure 6 where the vertices with the same label are in the same group. For any vertex (say abc), $\{abc, b\dot{c}\dot{a}, \dot{c}a\dot{b} \mid a,b,c \in \{1,2,3,\dot{1},\dot{2},\dot{3}\}\}$ must be in the same group in both cases. Without loss of generality, assume that vertex abc is in group1 together with vertices $b\dot{c}\dot{a}$ and $\dot{c}a\dot{b}$; $\dot{a}bc$ is in group2 together with vertices $\dot{c}\dot{a}\dot{b}$ and $b\dot{c}a$; $\dot{b}\dot{a}c$ is in group3 together with vertices $\dot{c}\dot{b}a$ and $\dot{a}\dot{c}b$; and $\dot{c}\dot{b}\dot{a}$ is in group4 together with vertices $\dot{b}ac$ and $a\dot{c}b$. Since vertex $\dot{a}bc$ in group2 is adjacent to $\dot{c}\dot{b}a$ in group3 and $\dot{b}ac$ in group4 and vertex $\dot{a}\dot{c}b$ in group3 is adjacent to $a\dot{c}b$ in group4, there are adjacent vertices in each pair of groups. Hence the labels for each part must be at least two apart, which implies $6 \le \lambda_{2,1}(BP_3)$. Therefore, $\lambda_{2,1}(BP_3) = 6$. ■

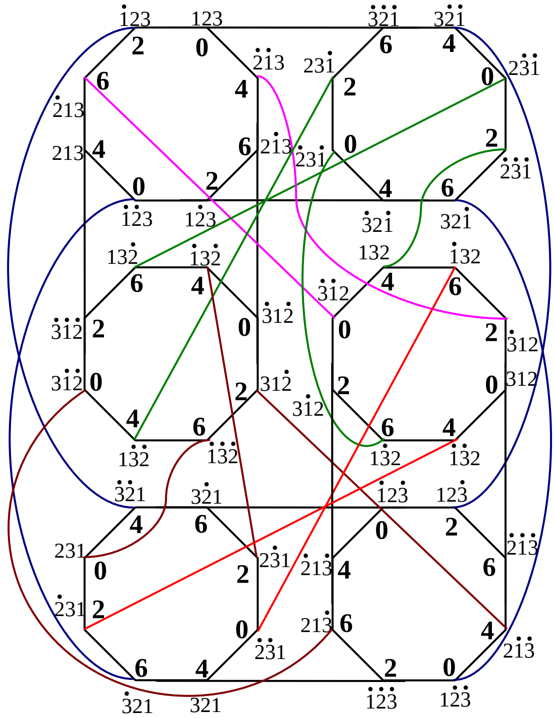

Figure 5. A 6-$L(2,1)$ labeling of BP_3 with abc and $a\dot{b}c$ are both in group1.

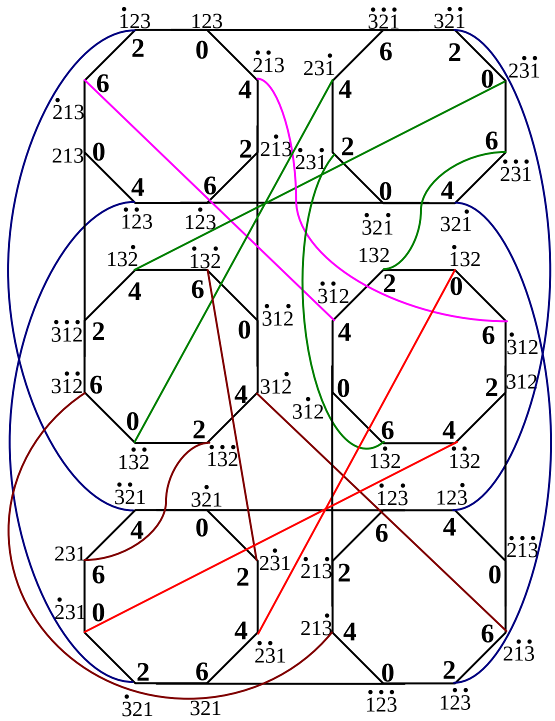

Figure 6. A 6-$L(2,1)$ labeling of BP_3 with abc and bac are both in group1.

Theorem 4. *Let BP_n be an n-dimensional Burnt Pancake graph. Then $n+2 \le \lambda(BP_n) \le 4n-2$ for $n \ge 4$.*

Proof: Consider a labeling $f : V(BP_n) \to \{0,2,4,\cdots,4n-2\}$ such that $f(v_i)=2i-2$ and $f(\dot{v}_i)=2n+2i-2$ where the vertex v starts with digit i and vertex $\dot{v}$ starts with digit $\dot{i}$ for $1 \le i \le n$ as shown in figure 4. By proposition 4, we know that each vertex with the same $L(2,1)$ label are at distance at least 3. Since only even numbers are used in f, if u and v are adjacent vertices, $|f(u)-f(v)| \ge 2$. Hence f is a $(4n-2)-L(2,1)$ labeling of BP_n, which implies $\lambda_{2,1}(BP_n) \le 4n-2$. By lemma 2, we have $n+2 \le \lambda_{2,1}(BP_n)$. Hence the result follows. ■

V. Folded Hypercube Graph

The *n*-dimensional folded hypercube graph FHC_n defined in [6] is an *n*-dimensional hypercube graph Q_n appended with 2^{n-1} complementary edges. The hypercube graph Q_n consists of 2^n vertices denoted distinctly by *n*-bit binary sequence numbers from 0 to 2^n-1 by $\{b_1b_2\cdots b_{n-1}b_n \mid b_i \in \{0,1\}\}$. Two vertices are adjacent in Q_n if their *n*-bit binary numbers differ in exactly one bit. The folded hypercube graph also contains 2^n vertices. A complementary edge means that vertex $u=\{u_1u_2\cdots u_{n-1}u_n \mid u_i \in \{0,1\}\}$ is adjacent to vertex $v=\{v_1v_2\cdots v_{n-1}v_n \mid v_i \in \{0,1\}\}$ where $u_i \ne v_i$ for $1 \le i \le n$. The edges in FHC_n contain $E(Q_n)$ and complementary edges which connect two farthest vertices in Q_n. A 3-folded hypercube graph which is shown in figure 7, contains 8 vertices; vertex 000 is adjacent with vertices 001, 010, 100, and 111. FHC_n is a (*n*+1) regular graph. Following propositions are useful in the proof of theorem 5.

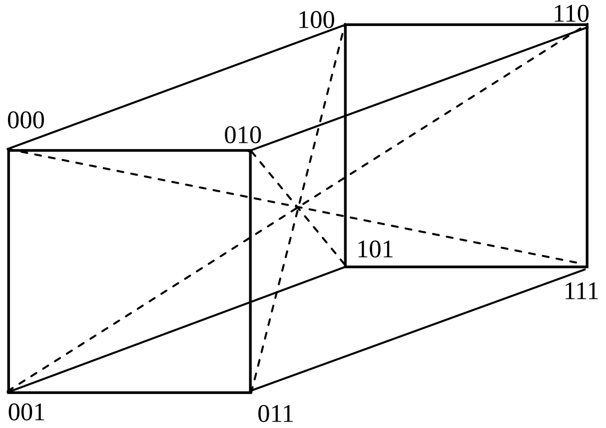

Figure 7. A 3-folded hypercube graph.

Proposition 5. [6] *The diameter of FHC_n is $\lceil \frac{n}{2} \rceil$.*

Proposition 6. [11] *FHC_n is a bipartite graph if and only if n is odd.*

For an n-dimensional folded hypercube graph FHC_n, since $FHC_1 = K_2$, $FHC_2 = K_4$, and $FHC_3 = K_{4,4}$, by proposition 2, we have $\lambda_{2,1}(FHC_1)=2$, $\lambda_{2,1}(FHC_2)=6$, and $\lambda_{2,1}(FHC_3)=8$. For the case of $n \ge 4$, we have following two theorems.

Theorem 5. *Let FHC_n be an n-dimensional Folded Hypercube graph. For $n=4$ or 5, $\lambda_{2,1}(FHC_4)=\lambda_{2,1}(FHC_5)=15$.*

Proof: By proposition 5, every vertex in $\lambda_{2,1}(FHC_4) \ge 2^4-1=15$. A 15-$L(2,1)$ labeling FHC_4 must have distinct labels, hence of FHC_4 is shown in figure 8, therefore $\lambda_{2,1}(FHC_4)=15$. By proposition 6, FHC_5 is a bipartite graph such that each partite set contains 2^4 vertices. Since the vertices in the same partite set of a bipartite graph must have even distance and by proposition 5, the diameter of FHC_5 is 3, the vertices in the same partite set are at distance two. Hence every vertex in the same partite set of FHC_5 must have distinct labels. That is $\lambda_{2,1}(FHC_5) \ge 2^4-1=15$. A 15-$L(2,1)$ labeling of FHC_5 is shown in table

1, therefore $\lambda_{2,1}(FHC_5)=15$. ■

Lemma 4. [9] *Let* Q_n *be the n-dimensional hypercube graph. Then for* $n\ge 5$, $\lambda_{2,1}(Q_n)\ge n+3$.

Lemma 5. [10] *Let* Q_n *be the n-dimensional hypercube graph. Then for all* n, $\lambda_{2,1}(Q_n)\le 2n$.

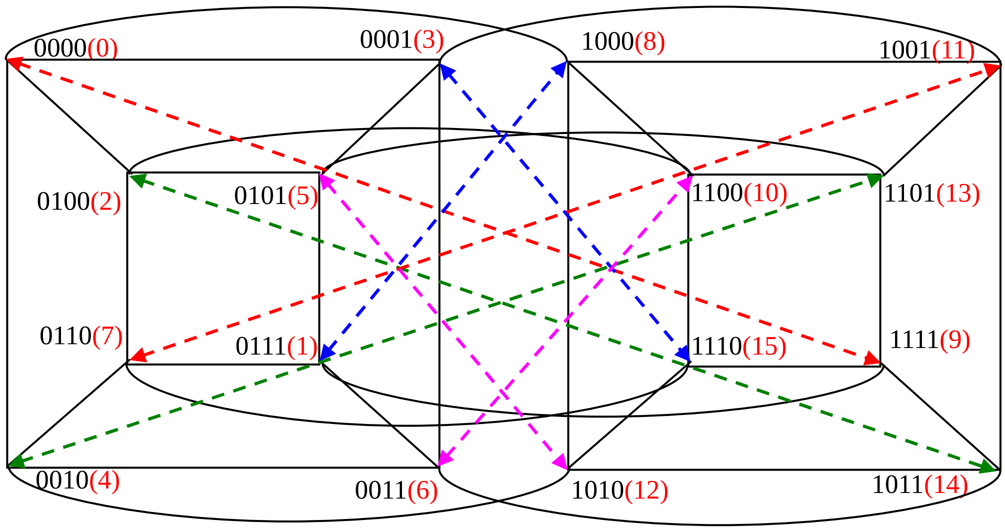

Figure 8. A 15-$L(2,1)$ labeling of FHC_4 (vertex (labeling number)).

Table 1. A 15-$L(2,1)$ labeling of FHC_5.

Partite set 1	vertex	00000	00011	00101	01001	10001	00110	01010	10010
	label	***0***	***1***	***2***	***3***	***4***	***5***	***6***	***7***
	vertex	01100	10100	11000	11110	11101	11011	10111	01111
	label	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***
Partite set 2	vertex	11111	00001	00010	00100	01000	10000	11100	11001
	label	***9***	***14***	***3***	***15***	***12***	***13***	***6***	***7***
	vertex	11010	10110	10101	10011	01110	01101	01011	00111
	label	***4***	***1***	***0***	***10***	***2***	***5***	***11***	***8***

Theorem 6. *Let* FHC_n *be an n-dimensional Folded Hypercube graph. Then* $n+3\le\lambda_{2,1}(FHC_n)\le 4n-2$ *for* $n\ge 6$.

Proof: Let $v\in V(FHC_n)$, $v=(v_1v_2\cdots v_n)$ where $v_i\in\{0,1\}$. Let $V(FHC_n)=V(Q_{n-1}^0)\cup V(Q_{n-1}^1)$ such that $V(Q_{n-1}^0)$ is the set of vertices with sequence number start from 0 and $V(Q_{n-1}^1)$ is the set of vertices with sequence number start from 1. By lemm5, there is a $L(2,1)$ labeling $f_0:V(Q_{n-1}^0)\to\{0,1,2,\cdots,2n-2\}$. Define $L(2,1)$ labeling $f:V(FHC_n)\to\{0,1,2,\cdots,4n-2\}$ by $f(v^0)=f_0(v^0)$ and $f(v^1)=f_0(v^0)+2n$ where

$v^0 \in V\left(Q_{n-1}^0\right)$ and $v^1 \in V\left(Q_{n-1}^1\right)$ are differ only at first bit in FHC_n. Since the labels of any two vertices u,v such that $u \in V\left(Q_{n-1}^0\right)$ and $v \in V\left(Q_{n-1}^1\right)$ are differ by at least two, and f_0 is a $L(2,1)$ labeling of Q_{n-1}, for any two vertices $u,v \in V(FHC_n)$, we have $|f(u)-f(v)| \geq 2$ if $(u,v) \in E(FHC_n)$ and $|f(u)-f(v)| \geq 1$ if $d(u,v)=2$. Hence f is a $(4n-2)-L(2,1)$ labeling of FHC_n, which implies $\lambda_{2,1}(FHC_n) \leq 4n-2$. Since Q_n is the spanning subgraph of FHC_n, by lemma 1 and lemma 4 we have $n+3 \leq \lambda_{2,1}(Q_n) \leq \lambda_{2,1}(FHC_n)$. That completes the proof. ■

VI. Conclusion

This paper deals with $L(2,1)$ labeling for several n-dimensional regular graphs. For star-like graph S_n, we gave exact results for $n \leq 4$ and both upper and lower bounds for $n \geq 5$, where the upper bound is about twice as the lower bound. For pancake graph PC_n, we gave exact results for $n \leq 3$ and both upper and lower bounds for $n \geq 4$, where the upper bound is about twice as the lower bound. For burnt pancake graph BP_n, we gave exact results for $n \leq 3$ and bounds for $n \geq 4$. Although the upper bound is about four times as the lower bound, we conjecture that $\lambda_{2,1}(BP_n) \geq 2n$, which makes the upper bound twice as the lower bound. For folded hypercube graph FHC_n, we gave exact results for $n \leq 5$ and both upper and lower bounds for $n \geq 6$.

Reference

[1]. S. B. Akers, D. Horel, and B. Krishnamurthy, “The star graph: an attractive alternative to the n-cube,” Proceedings of the International Conference on Parallel Processing, pp.393-400, 1987.

[2]. S. B. Akers and B. Krishnamurthy, “A group-theoretic model for symmetric interconnection networks,” IEEE Transactions on Computers, Vol. 38(4), pp.555-566, 1989.

[3]. T. Calamoneri, “The $L(h,k)$ -Labeling Problem-A Survey and Annotated Bibliography,” The Computer Journal, Vol. 49(5), pp. 585-608, 2006.

[4]. T. Calamoneri, “The $L(h,k)$ -Labeling Problem-A Survey and Annotated Bibliography,” 2009, available at: http://www.dsi.uniroma1.it/~calamo/PDF-FILES/survey.pdf .

[5]. D. S. Cohen, M. Blum, “On the problem of sorting burnt pancakes,” Discrete Applied Mathematics, Vol. 61, pp. 105-120, 1995.

[6]. A. El-Amawy, S. Latifi, “Properties and performance of folded hypercubes,” IEEE Trans. Parallel Distrib. Syst., Vol. 2(1), pp. 31–42, 1991.

[7]. J. R. Griggs, R. K. Yeh, “Labeling graphs with a condition at distance 2,” SIAM J. Discrete Mathematics, Vol. 5, pp. 586-595, 1992.

[8]. W. K. Hale, “Frequency assignment: theory and applications,” proc. IEEE, Vol. 68, pp. 1497-1514, 1980.

[9]. K. Jonas, “Graph colorings analogues with a condition at distance two: $L(2,1)$ -labelings and list λ -labelings,” Ph. D. Thesis, Department of Mathematics University of South Carolina, Columbia, SC, USA, 1993.

[10]. M. A. Whittlesey, J. P. Georges, D. W. Mauro, “On the λ -number of Q_n and related graphs,” SIAM J. Discrete Mathematics, Vol. 8, pp. 499-506, 1995.

[11]. J. M. Xu, M. Ma, “Cycles in folded hypercubes,” Applied Mathematics Letters, Vol. 19, pp. 140-145, 2006.

[12]. R. K. Yeh, “Labeling graphs with a condition at distance two,” Ph.D. thesis, Department of Mathematics, University of South Carolina, Columbia, SC, 1990.

[13]. R. K. Yeh, “A survey on labeling graph with a condition at distance two,” Discrete Mathematics, Vol. 306, pp. 1217-1231, 2006.