Fractal Compression on Mammograms

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Abstract

Mammography has been an effective and simple diagnostic method to detect the breast cancer in an early stage. In general, the suspicious region of the breast cancer or other abnormal symptom of breast occupies a bitty part of a mammogram. It takes 4MB storage space to archive the sampled part at spatial resolution 2048 X 2048 with 256 gray intensity. It is difficult to handle all patient mammograms in a picture archives and communication system for its incredible amount of storage space. Since the cloud-like tissue background of a mammogram almost appears the property of self similarity, the fractal compression technique is considered as a better solution to reduce its storage space. The results of the experiments show that it can cut down about 75% storage space for a block size of 4 X 4 and preserve image quality of average 35 dB Peak Signal to Noise Ratio (PSNR).

[Key words] fractal compression, mammogram, self similarity.

I. Introduction

In recent years, people have become familiar with fractal pictures in taking any scale magnification, the pictures are still looked infinitely similar [1]. Fractal pictures possess an infinite amount of self similarity in every subtle part as the structure of the whole[2]. The property of self similarity has been studied to illustrate the fractal geometry by Koch curve, Julia set, the coastline, and Mandelbrot set [2,3,4]. Generally, Euclidean geometry models an artificial object via regular shapes such as lines, circles, ellipses, squares, triangles, spheres and the notation of length, area, volume and curvature. Since fractal geometry can define a fractal dimension to describe the complexity of a natural object, therefore, it is suitable for the application to the unpredictable weather system, the irregular shapes of mountains and clouds, and the instability of heart beating [1,2,3,5,6,7]. Up to now, fractal geometry has been a newly mathematical discipline for inventing the disorder of nature to solve the unpredictable and nonlinear problems.

Fractal geometry is an astonishing and potential language used to describe the real world. Thus, several scientists and researchers of computer graphics have successfully generated and simulated by the fractal algorithms to the natures including leaves, trees, ferns, cloudy, rocks and mountains [2,3,5,6,8]. It also has been applied in several image applications such as image segmentation, image analysis and synthesis and image compression [9]. Contrast to generating a fractal pictures, Barnsley claimed the inverse problem of fractals to model real object [2,10,11]. Barnsley and Jacquin had realized fractal compression of an image that are approximated by combinations of iterated function system (IFS) code on some small blocks of an image [2,12,13,14,15].

Most recently, the breast cancer has risen to be the second leading factor of the women's cancer in the region of America and Taiwan [16,17]. The early detection of the breast cancer leads to successfully treatment, low cost treatment and reduction of mortality. So far, mammography screening still remains the promising technique for the early breast cancer detection. Thus, doctors recommend to those women whose ages are over 45 should take a mammography examination routine to prevent the breast cancer each year [18,19,20]. Picture archives and communication system is a logistic system to aid physicians to retrieve patient records with digitized images including ultra-sound image, computer tomography image (CT), Magnetic Resonance Image (MRI) and mammograms. Each mammogram is allocated in 4MB storage space to process at spatial resolution 2048 X 2048 with 256 gray level. Thus, it is a big job and quite a large space is also required. Since the cloud-like tissue background of a mammogram almost appears apparently the property of self similarity, the fractal compression technique is evaluated valuably to cut down the vast amount of storage space on a mammogram study.

II. Mathematical Background of Fractal Compression

Fractal dimension distinguishes from the naive topology dimension. The latter is always integer and is applicable in Euclidean space. However, the former may be fraction, even irrational. Fractal dimension is a metric parameter to measure self similarity of an object. For an N dimension space, an affine transformation S: $R^n \longrightarrow R^n$ is a similarity if there exists a constant c such that the distance measure $| S(x) - S(y) | = c | x-y |$ for all x, y in R^n [15]. As a result, even if an object is translated, zoomed, rotated, flipped, sheared and scaled, the similarity of the resulting object remains the same. Based on the affine transformation, Barnsley demonstrated an iterated function system to manipulate the generation of Barnsley leaf [10,11]. Barnsley is the first person who claims the fractal image compression application to commercialization [2]. A typical image, such as landscape, face and fern have partial copies of itself to present the whole one. Therefore, finding a local self similarity of an image is a successful way to encode an image in fractal technique [2,12].

In fractal image compression, it is necessary to find a fractal attractor to represent a part of image in a desired closure. The fractal attractor of an image can be obtained by some foundations of fractal image compression which are described as below.

(1) Affine transformation [2,11]

Definition 1: Affine transformation $W : R^2 \longrightarrow R^2$ is transformation of the form

$$W(X) = W\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} k & l \\ m & n \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} i \\ j \end{bmatrix} \quad (1)$$
$$= A X + t$$

where k, l, m, n, i and j are real number. Matrix A is a linear transformation that deforms an object through the form of scaling, rotation, mirror, flipping and shearing. Vector t is used to translate the center of an object.

(2) Contractive Mapping Theorem [11]

Definition 2: A transformation $W: X \longrightarrow X$ is contractive mapping on a Hausdorff metric space (X, d) if there is a constant c, $0 \le c \le 1$, such that the inequality

$$d(W(x), W(y)) \le c \cdot d(x, y) \quad (2)$$

is true for all x, y in X.

Let $W : X \longrightarrow X$ be a contractive mapping on a Hausdorff metric space (X, d). Then W possesses a unique and fixed point (fractal attractor) x_f, $x_f \in X$, such that a series of transformation W on x, in other words, the sequences $\{ W^{(n)}(x) : n=0, 1, 2, \ldots ; x \in X \}$ converges to x_f. That is

$$\lim_{n \to \infty} W^{(n)}(x) = x_f \quad (3)$$

The contractive mapping theorem implies that the detail of an object can be reconstructed if the fractal attractor and its affine transformations are obtained. This is why the compression ratio of fractal compression is set to high value.

(3) Iterated Function System [4]

Let $p_1, p_2, p_3, \ldots, p_n$ be points in R^2. For any point $p \in R^2$, let $W_i(p) = c \cdot (p - p_i) + p_i$, i= 1,..., n, and $0 \le c \le 1$. The collection of functions $\{W_1, W_2, \ldots W_n\}$ is called an iterated function system. If an initial points p_i of image and its transformations $W_i(p)$ are given, the fractal image can be produced by recursively computing the IFS code. This IFS code is very similar to multiple reduction copy machine metaphors [2].

(4) Collage Theorem [11, 12].

A typical camera-captured image is unlike to those with self similarity of Sierpinkski triangle and Koch curve types which are fractal wholly in everywhere. But, in fact, a part of an image may be similar to other part of itself. Actually, the image does contain a different sort of self similarity. Although an image can be recovered from copies of parts by itself, these parts are not the identical copies of themselves under affine transformation. Thus, we choose a closed copies of themselves that minimize the error between the original image and the approximated one.

Let (X, d) be a complete Hausdorff metric space. Let original image T belong to H(X), and let $\varepsilon \ge 0$ be given. Choose an IFS code $\{ X; \; W_1, W_2, \ldots, W_N \}$ with contractive factor c, $0 \le c \le 1$, so that

$$d\left(T, \bigcup_{i=1}^{N} W_i(T)\right) \le \varepsilon,$$

where d ( ) is Hausdorff metric. Then

$$d(T, A) \le (1-c)^{-1} \cdot d\left(T, \bigcup_{i=1}^{N} W_i(T)\right)$$

for all $T \in H(X)$ (4)

where A is the IFS attractor of T. We can find an IFS attractor close to a given image by seeking a set of contractive mappings on a suitable space according to the theorem. From the given image lies space, the union of the recovered image under the similar affine transformation is near to the original image.

III. The fractal compression Procedures

According to Collage Theorem, it is essential to partition an image into small pieces (range cells), then these pieces are considered as fractal attractors of parts (domain cells) of image. However, it is difficult to find the contractive factor of fractal attractor. For simplicity, the size of domain cell verse one of range cell is integer. As a range cell searches all domain cells, we need to choose the best candidate cell with minimal root mean square (RMS) error since it can generate this small fractal cell. Meanwhile the process of minimizing RMS error equation is proceeding, the contrast parameter and brightness offset for similar affine transformation are derived. The result is surprisingly satisfied. Encoding an image by fractal compression is just to record the similar affine transformation. The procedures of fractal image compression is listed as following [2,9,21].

(1) Range Block partition: This partition will divide an image into more small and not overlapped blocks. The shape of block may be square, rectangle and triangle. Their sizes are often chosen as 4 X 4, 8 X 8 or 16 X 16. The smaller the size is, the more the amount of block is. In general, a square block with size 4 X 4 is feasible to obtain a high fidelity of image quality. Each range block is recognized as a single IFS attractor, and is also named as range cell.

(2) Domain Block Choice: According to Contractive Mapping Theorem and the discussed similarity measure in later paragraph, a range cell should correspond to a domain cell from a pool of large scale of domain blocks under similar affine transformation. The size of a domain block can be any scale of range cell and they can be overlapped and sampled by pixel and pixel. For simplicity and quality evaluation, the size of domain block is double one of range cell and the movement is one half of the size of block.

(3) Block Shrinking: The similarity measure between domain cell and range cell must be done at the same size. Therefore, the domain cell must be downsampled by nearest neighborhood or interpolation.

(4) Isometrics: All images generated by taking mirror, flip, and rotation operations on an image are isometric in fractal compression. Thus, eight isometrics are shown in Figure 1 and they are considered isomorphic when similar affine transformation is performed.

(5) Search strategy: The performance of encoding procedure is absolutely dependent on the search strategy of domain cells. The popular search methods include fully exhaustive search, modified 2D-Logarithm search, conjugate direction search, One-at-a-Time search, 3-step search and classified search.

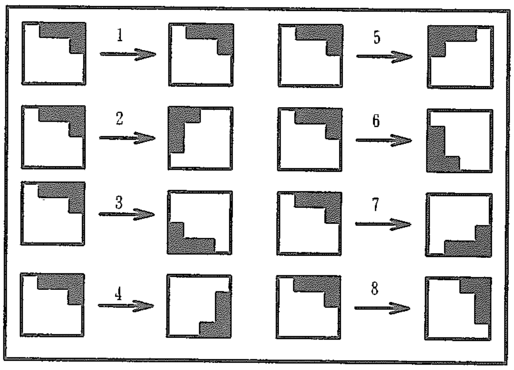
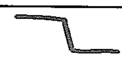


Figure 1. Isometries

Table 1 complexity classification

Class	Standard Deviation	Zero Crossings	Representation
Flat cell	less (< 1.0)	don't care	
Ballast cell	small (1.0 ~ 30.0)	large	
Step cell	small (1.0 ~ 30.0)	small	
Rock cell	large (> 30.0)	large	
Palisade cell	large (> 30.0)	small	

In this study, a complexity classification is introduced and the roughness of cell as a similarity measure is proposed to get a newly effective search strategy [21]. The roughness consist of the standard deviation σ of a cell, zero crossings ZC_x of a cell along horizontal direction and zero crossings ZC_y of a cell along vertical direction. The standard deviation σ of a cell can render the variation of gray level of a block in one dimension space. The zero crossings of a cell along one direction are defined as the change of pixel intensity of a line crossing over the mean of all pixels of a line in such a direction. Both zero crossings ZC_x and zero crossing ZC_y can reflect the variation of gray level of a block in two dimension space. According to the roughness of a block measured by the above two parameters, the complexity of a block can be classified into five categories which are shown in Table 1. Only these domain cells that belong to the same class of the range cell are the candidate cells for further comparison of self similarity.

(6) Self similarity measure and threshold: To evaluate the self similarity between a range cell and a domain cell, a criterion E that measures the difference between the range cell and the updated domain cell which has tuned its brightness and contrast is given as follow [2,12].

$$E = \sum_{j=1}^{N}\sum_{i=1}^{N} ((c*D_{ij} \ + b) - R_{ij})^2 / N^2 \qquad (5)$$

where R_{ij} is the pixel intensity of a range cell at row i and column j while D_{ij} is the pixel intensity of a domain cell at row i and column j, and the parameters b and c of equation (5) are brightness setting and contrast setting respectively. These parameters b and c can be determined by taking partial derivatives E with respect to b and c and let these equalities be zero. Finally, parameters b and c are obtained in following formulas [2,12].

$$c = \frac{\sum_{j=1}^{N}\sum_{i=1}^{N} R_{ij}*D_{ij} - \sum_{j=1}^{N}\sum_{i=1}^{N} R_{ij}*\sum_{j=1}^{N}\sum_{i=1}^{N} D_{ij}}{N^2*\sum_{j=1}^{N}\sum_{i=1}^{N} D_{ij}^2 - \left(\sum_{j=1}^{N}\sum_{i=1}^{N} D_{ij}\right)^2} \qquad (6)$$

$$b = \left(\sum_{j=1}^{N}\sum_{i=1}^{N} R_{ij} - c*\sum_{j=1}^{N}\sum_{i=1}^{N} D_{ij}\right) / N^2 \qquad (7)$$

If a domain cell is either with an error less than threshold T_e or with the least error among all domain cells, then it would be chosen as the best candidate cell close to the range cell. For speeding up the computation, the ratio of the standard deviation of range cell verse that of domain cell is used in this study to replace the parameter c in equation (6) with a little quality degradation.

(7) Encoding format: After parameters b and c are obtained, the encoding format of a range block of an image with 256 X 256 spatial resolution is encoded by 31 bytes that involves 8 bytes for horizontal position and 8 bytes for vertical position, 3 bytes for isometric index, and 6 bytes for contrast setting b and 6 bytes for brightness setting c [2].

IV. Encoding Algorithm

The encoding algorithm of fractal compression is shown in Figure 2. Before doing a progress of encoding a fractal cell, preprocessing first determines the mean, standard deviation and zero crossing of both range cells and domain cells. For each range cell,

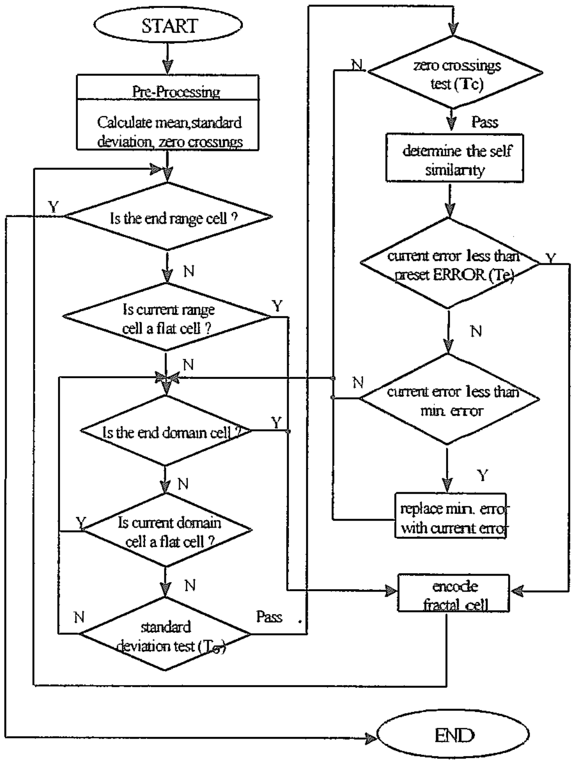


Figure 2. The flowchart of encoding algorithm

if it is a flat cell, its brightness setting is assigned to its mean, and its contrast parameter is zero. Otherwise, it will seek a domain cell that is not a flat cell from domain pool. The standard deviation test and the zero crossing test of complexity classification are used to filter out a group of domain cells that have the same manner of both the one dimension and the two dimension variations of current range cell. At standard deviation test phase, only those domain cells whose standard deviations are greater than T_σ times the one of range cell will do similarity measure. At zero crossing test phase, those domain cells whose zero crossings along X and Y direction fall in the interval of the zero crossings of range cell minus the threshold T_c are picked out to do similarity measure. Two cases can be considered as the best candidate cell when the domain cell with an error E of self similarity relates to current range cell is below the preset threshold T_e or the minimum one among all candidate domain cells. Then the relationship of the couple of best candidate cell and current range cell is registered in encoding format.

After a detailed analysis on our experimental images through fractal compression with and without using the complexity classification, the three thresholds T_σ, T_c and T_e are set to 1.3, 20 and 2 times the size of a block respectively for the following studies.

V. Mammogram Characteristics and Experiments

Observing a full screen of mammogram example as shown in Figure 3, it can be found that the mammogram consists of two parts. One is the exterior part of the breast with meaningless in diagnosis of some symptoms of breast; other is the inner part of the breast and it can be eventually divided into tissue background and the region of blood vessel, microcalifications and tumors. Doctors are always interesting in the inner part of breast for judging the possible disease of breast. The full screen of any mammogram is nearly full of fractal characteristics in anywhere, especially at exterior part breast and tissue background of breast. Thus, it is worth to study fractal compression on a mammogram.

In this study, ten mammographic images with size of 256*256 pixels at sample distance 0.1 mm and gray intensities of 8 bits are chosen. These mammographic images include 4 images with microcalcifications, 4 images with cloud-like tissue background and 2 images with the edge of breast. The experiments are proceeded in 486-DX4 platform with 16MB RAM. The Results of our experiments are shown in Table 2. As the image is encoded at the block size of 16 X 16, the average time and image quality of an image are 6.2 minutes and 33.60dB at compression ratio 0.0151 bits per pixel (bbp). As the image is encoded at the block size of 8 X 8, the average time and image quality of an image are 31.7 minutes and 34.85dB at compression ratio 0.0605 bbp. As the image is encoded at the block size of 4 X 4, the average time and image quality of an image are 129.8 minutes and 34.85dB at compression ratio 0.242 bbp. Figures 4 and figure 5 show typical examples of original mammograms and their reconstructed mammograms in this study. Although the differences of image quality among different block sizes are not separated in a few order, the subjective vision effect of an eye on the reconstructed mammograms can be clearly different. Finally, the block size of 4 X 4 is chosen as the best of all.

VI. Conclusions

Mammogram screen program is an important method to the early detection of breast cancer and teaching media in teaching field of mammography. To preserve the large amount of mammograms for further retrieve in previous two cases, the development of the digitilization of mammograms is necessary in Hospital Information Management System. In addition, all image processings such as enhancement, amplification, alignment, comparison,

Table 2. The experimental results

Block size / Image index	Time, PSNR (min., dB)	Time, PSNR (min., dB)	Time, PSNR (min., dB)
	16 X 16	8 X 8	4 X 4
#1	4.5, 32.63	23.4, 34.66	109.5, 36.16
#2	2.2, 36.67	14.8, 38.02	29.2, 35.98
#3	3.2, 33.72	23.4, 35.09	104.8, 38.02
#4	3.6, 34.52	23.1, 35.42	61.9, 34.58
#5	2.2, 36.10	4.5, 36.10	64.6, 36.23
#6	8.1, 34.76	64.5, 35.96	80.6, 34.22
#7	5.0, 35.21	37.2, 36.66	113.0, 34.64
#8	4.2, 31.57	27.9, 33.01	165.5, 33.92
#9	15.5, 28.68	75.9, 30.07	466.9, 29.39
#10	13.8, 32.23	22.9, 33.58	102.1, 35.36
Average	6.2, 33.60	31.7, 34.85	129.8, 34.85

long term archives, and picture transmission via networking issue the necessity of digital image.

In this study, ten different tissue parts of mammograms are chosen to evaluate the feasible of fractal compression on mammograms. We proposed a complexity classification search method to determine the roughness of a block and promote the performance of encoding algorithm. The subjective vision evaluations of our experiments shows that encoding the mammograms at the block size of 4 X 4 could obtain a better result of compression ratio. 0.242 bbp and image quality 34.85dB.

In summary, the study of fractal compression on mammograms seems promising, but it may combine other detection techniques on suspicious region of a mammogram to meet the requirement of high fidelity of image quality in medical images. The suspicious regions of either breast cancer or other abnormal symptom of a mammogram are encoded in lossless format while other parts are encoded in fractal compression format. Thus, high compression ratio and high fidelity of an image can be done by using mixed compression methods to encode different parts of a mammogram.

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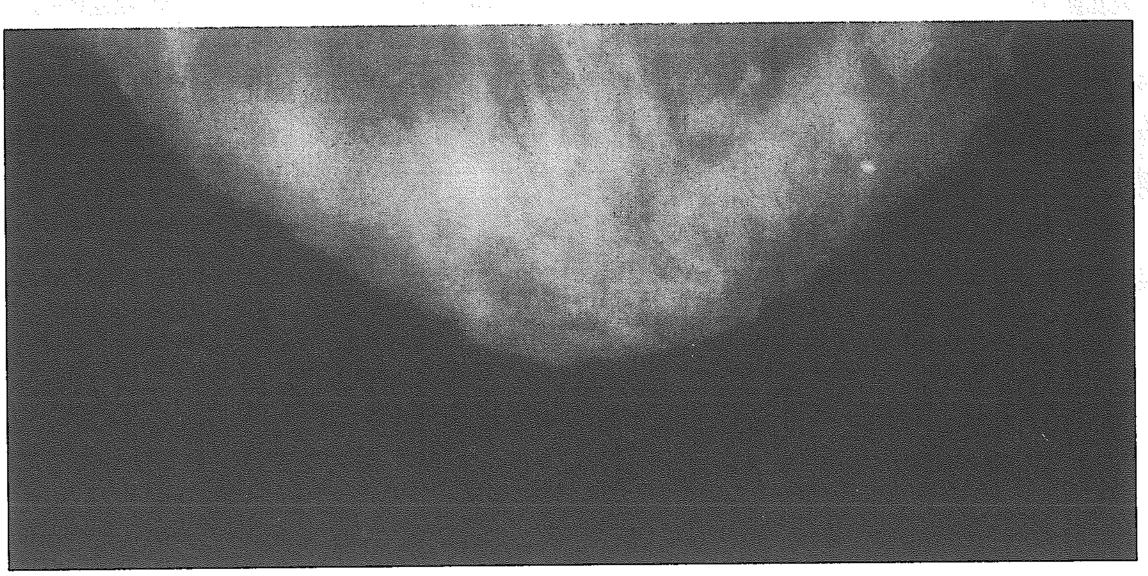

Figure 3. A typical mammogram.

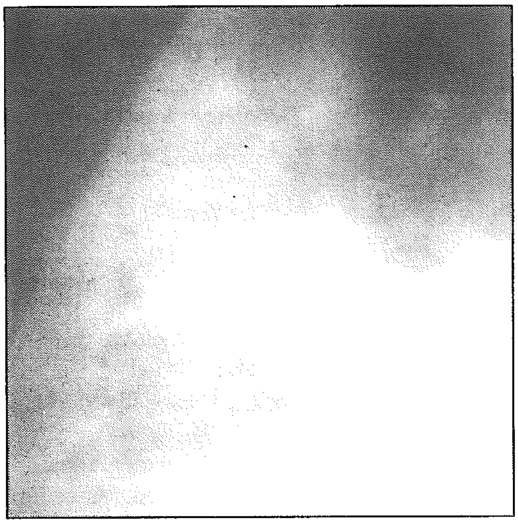

Figure 4.A. Original mammogram with index #3.

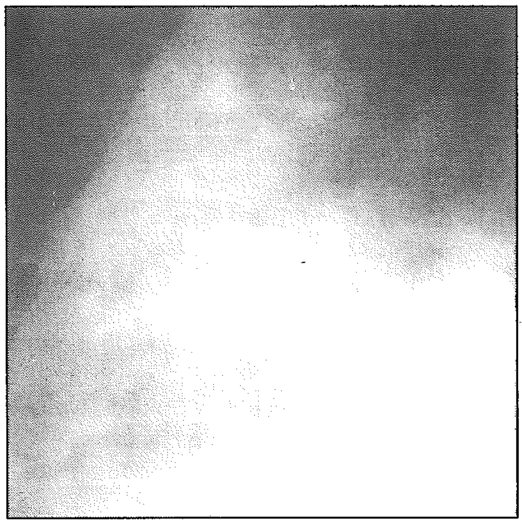

Figure 4.B. Reconstructed mammogram with size 16 X 16 and 0.0151 bpp.

Figure 4.C. Reconstructed mammogram with size 8 X 8 and 0.0605 bbp.

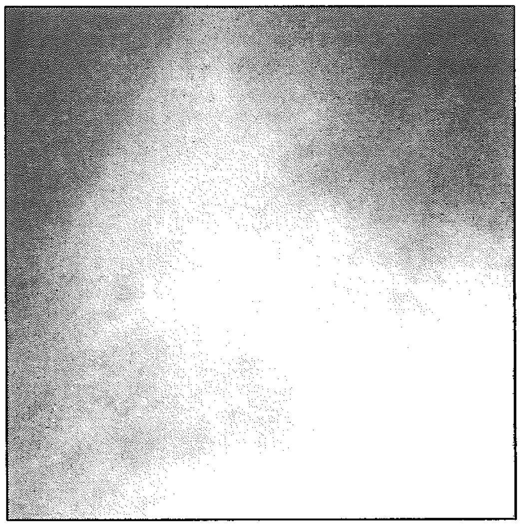

Figure 4.D. Reconstructed mammogram with size 4 X 4 and 0.242 bbp.

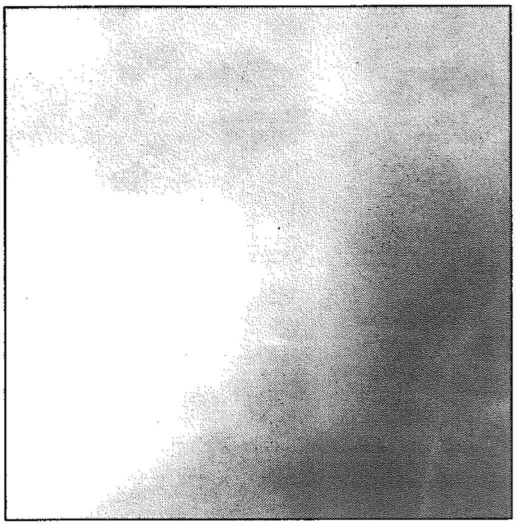

Figure 5.A. Original mammogram with index #6.

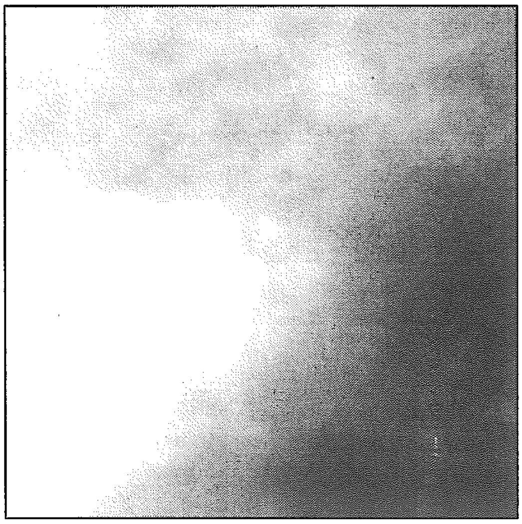

Figure 5.B. Reconstructed mammogram with size 16 X 16 and 0.0151 bpp.

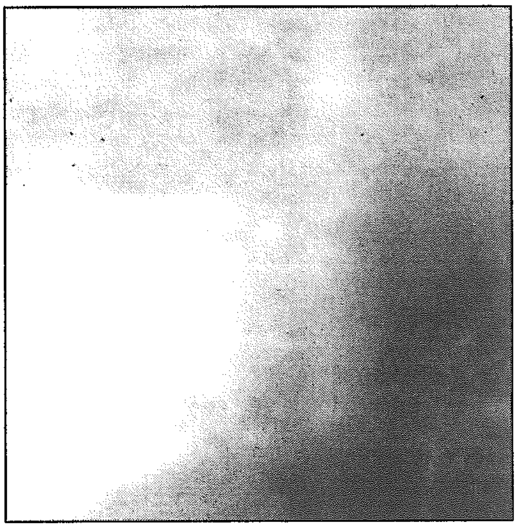

Figure 5.C. Reconstructed mammogram with size 8 X 8 and 0.0605 bbp.

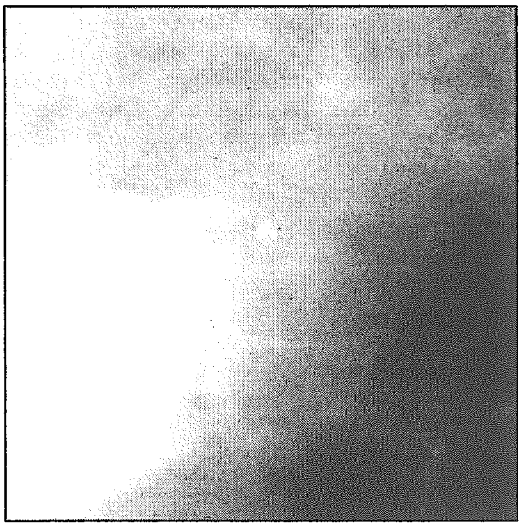

Figure 5.D. Reconstructed mammogram with size 4 X 4 and 0.242 bbp.