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New Methods for Evaluating Students' Answerscripts Using Fuzzy Sets

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Abstract

In this paper, we propose two new methods for evaluating students' answerscripts. The proposed methods can overcome the drawbacks of the ones presented in [2] due to the fact that they don't need to perform the complicated matching operations and they can evaluate students' answerscripts in a more fair manner.

Keywords: Fuzzy Grade Sheet, Fuzzy Set, Matching Function, Satisfication Level, Students' Answerscripts Evaluation.

1. Introduction

In recent years, some research on the application of fuzzy set theory [15] in education has begun [2], [4], [10]. In [10], Chiang et al. presented a method for the application of fuzzy set theory to teaching assessment. In [4], Chang et al. presented a method for fuzzy assessment of learning performance of junior school students. In [2], Biswas pointed out that the chief aim of education institutions should be to provide the students with the evaluation reports regarding their test/examination as sufficient as possible and with unavoidable error as small as possible. He also presented a fuzzy evaluation method (fem) for the application of fuzzy sets in students' answerscripts evaluation. The fem method presented in [2] is a computer based fuzzy approach, where a vector valued marking is used. Furthermore, in [2], Biswas also generalized the fem method to propose a generalized fuzzy evaluation method (gfem) in which a matrix-valued marking is adopted. However, the methods presented in [2] have the following drawbacks:

1) Because they use a matching function S to measure the degrees of similarity between the standard fuzzy sets and the fuzzy marks of the questions, they will take a large amount of time to perform the matching operations.

2) In Biswas's methods, two different fuzzy marks may be translated into the same awarded grade and this is unfair in students' evaluation.

Because Biswas's methods have the above two drawbacks in the task of students' answerscripts evaluation, it is necessary to develop new methods to overcome the above drawbacks.

In this paper, we present two new methods for the application of fuzzy sets in students' answerscripts evaluation. They can overcome the drawbacks of the ones presented in [2]. The proposed methods gave the advantages of much faster execution and are more fair in the task of students' evaluation than the ones presented in [2].

This paper is organized as follows. In Section 2, we briefly review the theory of fuzzy sets from [5], [6], [12], [15], and [16]. In Section 3, we briefly review Biswas's methods for students' answerscripts evaluation. In Section 4, we present two new methods for students' answerscripts evaluation using fuzzy sets. The conclusions are discussed in Section 5.

2. Fuzzy Set Theory

In [15], Zadeh proposed the theory of fuzzy sets. Roughly speaking, a fuzzy set is a class with fuzzy boundaries. Let X be the universe of discourse, $X = \{x_1, x_2, \ldots, x_n\}$, and let S be a fuzzy set of X, then the fuzzy set A can be represented as:

$$A = \{(x_1, f_A(x_1)), (x_2, f_A(x_2)), \ldots, (x_n, f_A(x_n))\}, \quad (1)$$

where f_A is the membership function of the fuzzy set A, f_A: $X \rightarrow [0, 1]$, and $f_A(x_i)$ indicates the grade of membership of x_i in A. If the universe of discourse X is an infinite set, then the fuzzy set A can be expressed as:

$$A = \int_X f_A(x_i)/x_i,\ x_i \in X. \quad (2)$$

Example 2.1: Let X be the universe of discourse, X= {red, black, yellow, blue, white, brown, green}, and let "dark" be a fuzzy set of the universe of discourse X subjectively defined as follows:

dark = {(red, 0.5), (black, 1.0), (yellow, 0.1), (blue, 0.6), (white, 0.0), (brown, 0.8), (green, 0.3)}, (3)

where "black" has the largest membership value (i.e., 1.0) in the fuzzy set "dark", and "white" has the smallest membership value (i.e., 0.0) in the fuzzy set "dark". Thus, "black" is most pertinent to the fuzzy set "dark", and "white" is impertinent to the fuzzy set "dark".

For convenience, if an element x_i has zero membership value in a fuzzy set A (i.e., $f_A(x_i) = 0$), then the ordered pair $(x_i, f_A(x_i))$ can be discarded from the representation of the fuzzy set. Thus, in the above example, the fuzzy set "dark" also can be written as follows:

dark = {(red, 0.5), (black, 1.0), (yellow, 0.1), (blue, 0.6), (brown, 0.8), (green, 0.3)}. (4)

Example 2.2: Let X be the universe of discourse, X = [0, 100]. Then, the fuzzy sets "young" and "old" may subjectively be defined as follows:

$$f_{young}(x) = \begin{cases} 1, & 0 < x \leq 20 \\ (1 + ((x-20)/15)^2)^{-1}, & 20 < x \leq 100, \end{cases} \quad (5)$$

$$f_{old}(x) = \begin{cases} 0, & 0 < x \leq 40 \\ (1 + ((x-40)/15)^{-2})^{-1}, & 40 < x \leq 100, \end{cases} \quad (6)$$

where f_{young} and f_{old} are the membership functions of the fuzzy sets "young" and "old", respectively.

3. Biswas's Methods for Students' Answerscripts Evaluation

In [2], Biswas used a matching function S to measure the degree of similarity between two fuzzy sets. Let A and B be two fuzzy sets of the universe of discourse X, where

$A = \{(x_1, f_A(x_1)), (x_2, f_A(x_2)), \ldots, (x_n, f_A(x_n))\}$,
$B = \{(x_1, f_B(x_1)), (x_2, f_B(x_2)), \ldots, (x_n, f_B(x_n))\}$,
$X = \{x_1, x_2, \ldots, x_n\}$.

By using the vector representation method, the fuzzy sets A and B can be represented by the vectors $\overline{A}$ and $\overline{B}$, respectively, where

$\overline{A} = <(x_1, f_A(x_1)), (x_2, f_A(x_2)), \ldots, (x_n, f_A(x_n)>$,
$\overline{B} = <(x_1, f_B(x_1)), (x_2, f_B(x_2)), \ldots, (x_n, f_B(x_n)>$.

Then, the degree of similarity $S(\overline{A}, \overline{B})$ between the fuzzy sets A and B can be measured by

$$S(\overline{A}, \overline{B}) = \frac{\overline{A} \bullet \overline{B}}{Max(\overline{A} \bullet \overline{A}, \overline{B} \bullet \overline{B})}, \quad (7)$$

where $S(\overline{A}, \overline{B}) \in [0, 1]$. The larger the value of $S(\overline{A}, \overline{B})$, the more the similarity between the fuzzy sets A and B.

Based on the matching function S, Biswas et al. introduced a fuzzy evaluation method (fem) for evaluating students' answerscripts. In the following, we briefly review Biswas's methods for students' answerscripts evaluation. In [2], Biswas used five fuzzy linguistic hedges (called Standard Fuzzy Sets (SFS)) for students' answerscripts evaluation, i.e., E (excellent), V (very good), G (good), S (satisfactory), and U (unsatisfactory), where

X = {0%, 20%, 40%, 60%, 80%, 100%},
E = {(0%, 0), (20%, 0), (40%, 0.8), (60%, 0.9), (80%, 1), (100%, 1)},
V = {(0%, 0), (20%, 0), (40%, 0.8), (60%, 0.9), (80%, 0.9), (100%, 0.8)},
G = {(0%, 0), (20%, 0.1), (40%, 0.8), (60%, 0.9), (80%, 0.4), (100%, 0.2)},
S = {(0%, 0.4), (20%, 0.4), (40%, 0.9), (60%, 0.6), (80%, 0.2), (100%, 0)},
U = {(0%, 1), (20%, 1), (40%, 0.4), (60%, 0.2), (80%, 0), (100%, 0)}.

Based on the vector representation method, the fuzzy set E, V, G, S, and U can be represented by the vectors $\overline{E}$, $\overline{V}$, $\overline{G}$, $\overline{S}$, and $\overline{U}$, respectively, where

$\overline{E} = <0, 0, 0.8, 0.9, 1, 1>$,
$\overline{V} = <0, 0, 0.8, 0.9, 0.9, 0.8>$,
$\overline{G} = <0, 0.1, 0.8, 0.9, 0.4, 0.2>$,
$\overline{S} = <0.4, 0.4, 0.9, 0.6, 0.2, 0>$
$\overline{U} = <1, 1, 0.4, 0.2, 0, 0>$.

In [2], Biswas pointed out that "A", "B", "C", "D", and "E" are called letter grades, where

$0 \leq E < 30$,
$30 \leq D < 50$,
$50 \leq C < 70$,
$70 \leq B < 90$,
$90 \leq A \leq 100$.

Furthermore, he also introduced the concept of mid-grade-point, where the mid-grade-point of A = 95 is denoted by P(A), B = 80 by P(B), C = 60 by P(C), D = 40 by P(D), E = 15 by P(E). Assume that an evaluator is to evaluate the *i*th question (i.e., Q.i) of an answerscript of a student using a fuzzy grade sheet shown in Table 1. In the first row of Table 1, the fuzzy mark (fum) to the answer of question Q.1 shows the degrees of the evaluator's satisfication for that answer in 0%, 20%, 40%, 60%, 80%, and 100% are 0, 0.1, 0.2, 0.4, 0.4, and 0.6, respectively. Let the fuzzy mark of the answer of question Q.1 be denoted by F_1. Then, we can see that F_1 is a fuzzy set of the universe of discourse X, where

X = {0%, 20%, 40%, 60%, 80%, 100% },
F_1 = {(0%, 0), (20%, 0.1), (40%, 0.2), (60%, 0.4), (80%, 0.4), (100%, 0.6)}.

Biswas's algorithm [2] for students' answerscript evaluation is summarized as follows.

TABLE 1

A Fuzzy Grade Sheet

Question Number	Fuzzy Mark						Grade
Q.1	0	0.1	0.2	0.4	0.4	0.6	
Q.2							
Q.3							
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
			Total Mark =				

Step1: For each attempted question in the answerscript repeatedly perform the following steps:

(1) the evaluator awards a fuzzy mark F_i to the question Q.i by his best possible judgement and fills up the cells of the *i*th row for the first seven columns. Let $\overline{F_i}$ be the vector representation of F_i.

(2) Calculate the following degrees of similarities: $S(\overline{E}, \overline{F_i})$, $S(\overline{V}, \overline{F_i})$, $S(\overline{G}, \overline{F_i})$, $S(\overline{S}, \overline{F_i})$, and $S(\overline{U}, \overline{F_i})$, where $\overline{E}$, $\overline{V}$, $\overline{G}$, $\overline{S}$, and $\overline{U}$ are the vector representations of the standard fuzzy sets E (excellent), V (very good), G (good), S (satisfactory), and U (unsatisfactory), respectively.

(3) Find the maximum among the five values $S(\overline{E}, \overline{F_i})$, $S(\overline{V}, \overline{F_i})$, $S(\overline{G}, \overline{F_i})$, $S(\overline{S}, \overline{F_i})$, and $S(\overline{U}, \overline{F_i})$. Assume that $S(\overline{V}, \overline{F_i})$ is the maximum value among the values of $S(\overline{E}, \overline{F_i})$, $S(\overline{V}, \overline{F_i})$, $S(\overline{G}, \overline{F_i})$, $S(\overline{S}, \overline{F_i})$, and $S(\overline{U}, \overline{F_i})$, then award grade "B" to the question Q.i due to the fact that grade "B" corresponds to V (very good) of the standard fuzzy set.

Step 2: Calculate the total score using the following formula:

$$\text{Total score} = \frac{1}{100} \Sigma [T(Q.i) \times P(g_i)], \tag{8}$$

where T(Q.i) is the mark alloted to Q.i in the question paper, and g_i is the grade awarded to Q.i by Step 1 of the algorithm. Put this total score in the appropriate box at the bottom of the fuzzy grade sheet.

Furthermore, in [2], Biswas also presented a generalized fuzzy evaluation method (gfem), where a generalized fuzzy grade sheet shown in Table 2 is used to evaluate the students' answerscripts. In the grade sheet of Table 2, for all j = 1, 2, 3, 4, and for all i, g_{ij} is the calculated grade by fem for the awarded fum F_{ij}, and m_i is the calculated mark to be awarded to the attempted question Q.i using the formula:

$$m_i = \frac{1}{400} \times T(Q.i) \times \sum_{j=1}^{4} P(g_{ij}) \tag{9}$$

and Total mark = $\Sigma\, m_i$.

TABLE 2

A Generalized Fuzzy Grade Sheet

Question Number	gfum	Grade	Mark
Q.1	F_{11}	G_{11}	m_1
	F_{12}	G_{12}	
	F_{13}	G_{13}	
	F_{14}	G_{14}	
Q.2	F_{21}	G_{21}	m_2
	F_{22}	G_{22}	
	F_{23}	G_{23}	
	F_{24}	G_{24}	
...	...	...	...
...	...	...	...
...	...	...	...
		Total Mark =	

However, the methods presented in [2] have the following drawbacks:

(1) Because they use a matching function S to measure the degree of similarity between the standard fuzzy sets and the fuzzy marks of the questions, it will take a large amount of time to perform the matching operations. Especially, when the number of questions in the test/examination is very big.

(2) In Biswas's method, two different fuzzy marks may be translated into the same awarded grade and this is unfair in students evaluation. For example, let F_i and F_j be two different fuzzy marks represented by fuzzy sets of the universe of discourse X, respectively, and let E (excellent), V (very big), G (good), S (satisfactory), and U (unsatisfactory) be standard fuzzy sets of the universe of discourse X, where X = {0%, 20%, 40%, 60%, 80%, 100%} and the corresponding awarded grade of the

standard fuzzy sets "E", "V", "G", "S", and "U" are "A", "B", "C", "D", and "E", respectively. Then, based on [2], we can calculate the following degrees of similarities:

Case 1: If $S(\overline{V}, \overline{F}_i)$ is the maximum value among the values of $S(\overline{E}, \overline{F}_i)$, $S(\overline{V}, \overline{F}_i)$, $S(\overline{G}, \overline{F}_i)$, $S(\overline{S}, \overline{F}_i)$, $F(\overline{U}, \overline{F}_i)$, then the fuzzy mark F_i is translated to the awarded grade "B" due to the fact that the grade "B" corresponds to V (very big).

Case 2" If $S(\overline{V}, \overline{F}_j)$ is the maximum value among the values of $S(\overline{E}, \overline{F}_j)$, $S(\overline{V}, \overline{F}_j)$, $S(\overline{G}, \overline{F}_j)$, $S(\overline{S}, \overline{F}_j)$, $F(\overline{U}, \overline{F}_j)$, then the fuzzy mark F_j is translated to the awarded grade "B" due to the fact that the grade "B" corresponds to V (very big).

From Case 1 and Case 2, we can see that two different fuzzy marks F_i and F_j are translated to the same awarded grade "B", and this is unfair in the task of students' answerscripts evaluation.

Because Biswas's methods have the above two drawbacks in the task of students' answerscripts evaluation, new methods for students' answerscripts evaluation is required to overcome the above drawbacks.

4. New methods for student's evaluation using fuzzy sets

In this section, we present two new methods for students' answerscripts evaluation. Assume that there are eleven satisfication levels to evaluate the students' answerscripts regarding a question of a test/examination, i.e., extremely good (EG), very very good (VVG), very good (VG), good (G), more or less good (MG), fair (F), more or less bad (B), very bad (VB), very very bad (VVB), and extremely bad (EB), where the degrees of satisfication of the eleven satisfication levels are shown in Table 3.

Let X be a set of satisfication levels, X = {extremely good, very very good (VVG), very good (VG), good (G), more or less good (MG), fair (F), more or less bad (B), very bad (VB), very very bad (VVB), and extremely bad (EB)},and let T be a mapping function which maps a satisfication level to the maximum degree of satisfication of the corresponding satisfication level, where $T : X \rightarrow [0, 1]$. From Table 3, we can see that

$$\begin{aligned}
&T(\text{extremely good}) = 1.00 \text{ (i.e., } T(EG) = 1.00),\\
&T(\text{very very good}) = 0.99 \text{ (i.e., } T(VVG) = 0.99).\\
&T(\text{very good}) = 0.90 \text{ (i.e. } T(VG) = 0.90),\\
&T(\text{good}) = 0.80 \text{ (i.e., } T(G) = 0.80),\\
&T(\text{more or less good}) = 0.70 \text{ (i.e., } T(MG) = 0.70),\\
&T(\text{fair}) = 0.60 \text{ (i.e., } T(F) = 0.60),\\
&T(\text{more or less bad}) = 0.50 \text{ (i.e., } T(MB) = 0.50),\\
&T(\text{bad}) = 0.40 \text{ (i.e., } T(VB) = 0.40),\\
&T(\text{very bad}) = 0.24 \text{ (i.e., } T(VB) = 0.24),\\
&T(\text{very very bad}) = 0.09 \text{ (i.e., } T(VVB) = 0.09),\\
&T(\text{extremely bad}) = 0 \text{ (i.e., } T(EB) = 0).
\end{aligned} \tag{10}$$

TABLE 3
Satisfication Levels and Their Corresponding Degrees of Satisfication

Satisfication Levels	Degrees of Satisfication
extremely good (EG)	100% (i.e., 1.00)
very very good (VVG)	91%-99% (i.e. 0.91- 0.99)
very good (VG)	81%-90% (i.e. 0.81- 0.90)
good (G)	71%-80% (i.e. 0.71- 0.80)
more or less good (MG)	61%-70% (i.e. 0.61- 0.70)
fair (F)	51%-60% (i.e. 0.51- 0.60)
more or less bad (MB)	41%-50% (i.e. 0.41- 0.50)
bad (B)	25%-40% (i.e. 0.25- 0.40)
very bad (VB)	10%-24% (i.e. 0.10- 0.24)
very very bad (VVB)	1%-9% (i.e. 0.01- 0.09)
extremely bad (EB)	0% (i.e. 0)

Assume that an evaluator can evaluate the students' answerscripts using extended fuzzy grade sheets. The definition of the extended fuzzy grade sheets is presented as follows.

Definition 4.1 : Extended fuzzy grade sheet: An extended fuzzy grade sheet is a matrix type structure containing thirteen columns and n rows, where n is the total number of questions in a test/examination. An example of an extended fuzzy grade sheet is shown in Table 4. At the bottom of the sheet there is a box which tells the total score. The first column reveals the serial numbers of the questions; in any row, the columns from the second to the twelfth shows the fuzzy mark awarded to the answer to the corresponding question in the first column, where the fuzzy mark is represented as a fuzzy set in the universe of discourse X, X = {extremely good, very very good (VVG), very good (VG), good (G), more or less good (MG), fair (F), more or less bad (B), very bad (VB), very very bad (VVB), and extremely bad (EB)}, The last (i.e., the thirteenth) column shows the degree of satisfication evaluated by the proposed method awarded to each question. The box at the bottom shows the total mark awarded to the student.

For example, assume that an evaluator is using an extended fuzzy grade sheet to evaluate the fuzzy mark of the first question (i.e., Q.1) of a test/examination of a student as shown in Table 5. From Table 5, we can see that the satisfication level regarding the first question of the student's answerscript is represented by a fuzzy set F(Q.1) of the universe of discourse X, where X = {EG, VVG, VG, G, MG, F, MB, B, VB, VVB, EB}, and

TABLE 4
An Extended Fuzzy Grade Sheet

Question Number	Satisfication Levels											Degree of Satisfication
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
Q.1												
Q.2												
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.n												
											Total	Mark =

$$F(Q.1) = \{(EG, 0), (VVG, 0.9), (VG, 0.8), (G, 0.5), (MG, 0), (F, 0), (MB, 0), (B, 0), (VB, 0), (VVB, 0), (EB, 0)\}. \quad (11)$$

For convenience, the fuzzy set F(Q.1) can also be abbreviated into

$$F(Q.1) = \{(VVG, 0.9), (VG, 0.8), (G, 0.5)\}. \quad (12)$$

It indicates that the satisfication level of the student's answerscript with respect to the first question is described as 90% very very good, 80% very good, and 50% good.

TABLE 5
An Example of An Extended Fuzzy Grade Sheet

Question Number	Satisficaction Levels											Degree of Satisfication
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
Q.1	0	0.9	0.8	0.5	0	0	0	0	0	0	0	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
											Total	Mark =

The method for students' answerscripts evaluation is now presented as follows:

Step 1: Assume that the fuzzy mark of the question Q.i of student's answerscript evaluated by an evaluator is shown in Table 6, where $y_i \in [0, 1]$ and $1 \le i \le 11$. From formula (10), we can see that T(EG) = 1, T(VVG) = 0.99, T(VG) = 0.90, T(G) = 0.80, T(F) = 0.60, T(MB) = 0.50, T(B) = 0.40, T(VB) = 0.24, T(VVB) = 0.09, and T(EB) = 0. In this case, the degree of satisfication D(Q.i) of the question Q.i of the student's answerscript can be evaluated by the function D,

$$D(Q.i) = \frac{y_1 * T(EG) + y_2 * T(VVG) + \ldots + y_{11} * T(EB)}{y_1 + y_2 + \ldots + y_{11}}, \quad (13)$$

where $D(Q.i) \in [0, 1]$. The larger the value of D(Q.i), the more the degree of satisfication that the question Q.i of the student's answerscript satisfies the evaluator's opinion.

For example, let's consider the example shown in Table 5. From formula (10), we can see that T(VVG) = 0.99, T(VG) = 0.90 , and T(G) = 0.80. By applying formula (13), the degree of satisfication D(Q.1) of the student's answerscript regarding question Q.1 can be evaluated as follows:

$$D(Q.1) = \frac{0.9 * 0.99 + 0.8 * 0.90 + 0.5 * 0.80}{0.9 + 0.8 + 0.5} = 0.9141. \quad (14)$$

It indicates that the degree of satisfication of the question Q.1 of the student's answerscript evaluated by the evaluator is 0.9141 (i.e., 91.41%).

Step 2 : Consider a candidate's answerscript to a paper of 100 marks. Assume that in total there were n questions to be answered:

TOTAL MARKS = 100
Q.1 carries s_1 marks
Q.2 carries s_2 marks
⋮
Q.n carries s_n marks,

TABLE 6
Fuzzy Mark of Question Q.i in An Extended Fuzzy Grade Sheet

Question Number	Satisficaction Levels											Degree of Satisfication
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.i	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
											Total	Mark =

where $\sum s_i = 100$, $0 \le s_i \le 100$, $1 \le i \le n$. Assume that the evaluated degree of satisfication of the question Q.1, Q.2, ..., and Q.n are D(Q.1), D(Q.2), ..., and D(Q.n), respectively, then the total score of the student can be evaluated as follows:

$$s_1 * D(Q.1) + s_2 * D(Q.2) + \ldots + s_n * D(Q.n) . \quad (15)$$

Put this total score in the appropriate box at the bottom of the extended fuzzy grade sheet.

In the following, we use an example to illustrate the student's answerscript evaluation process.

Example 4.1: Consider a candidate's answerscript to a paper of 100 marks. Assume that in total there were four questions to be answered:

TOTAL MARKS = 100
Q.1 carries 20 marks
Q.2 carries 30 marks
Q.3 carries 25 marks
Q.4 carries 25 marks

Assume that an evaluator awards the student's answerscript by an extended fuzzy grade sheet as shown in Table 7.

TABLE 7
Extended Fuzzy Grade Sheet of Example 4.1

Question Number	Satisfication Levels											Degree of Satisfication
	EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB	
Q.1	0	0.8	0.9	0	0	0	0	0	0	0	0	0.9424
Q.2	0	0	0	0.6	0.9	0.5	0	0	0	0	0	0.7050
Q.3	0	0	0.8	0.7	0.5	0	0	0	0	0	0	0.8150
Q.4	0	0	0	0	0	0	0	0.5	0.9	0.2	0	0.2713
											Total	Mark = 67

[Step 1] Base on formula (10) and by applying formula (13), we can see that

$$D(Q.1) = \frac{0.8 * T(VVG) + 0.9 * T(VG)}{0.8 + 0.9} = \frac{0.8 * 0.99 + 0.9 * 0.90}{0.8 + 0.9} = 0.9424 \quad (16)$$

$$D(Q.2) = \frac{0.6 * T(G) + 0.9 * T(MG) + 0.5 * T(F)}{0.6 + 0.9 + 0.5} = \frac{0.6 * 0.80 + 0.9 * 0.70 + 0.5 * 0.60}{0.6 + 0.9 + 0.5} = 0.7050 \quad (17)$$

$$D(Q.3) = \frac{0.8 * T(VG) + 0.7 * T(G) + 0.5 * T(MG)}{0.8 + 0.7 + 0.5}$$

$$= \frac{0.8 * 0.90 + 0.7 * 0.80 + 0.5 * 0.70}{0.8 + 0.7 + 0.5}$$
$$= 0.8150 \quad (18)$$

$$D(Q.4) = \frac{0.5 * T(B) + 0.9 * T(VB) + 0.2 * T(VVB)}{0.5 + 0.9 + 0.2}$$
$$= \frac{0.5 * 0.40 + 0.9 * 0.24 + 0.2 * 0.09}{0.5 + 0.9 + 0.2}$$
$$= 0.2713 \quad (19)$$

[Step 2] By applying formula (15), the total mark of the student can be evaluated as follows:

$$20 * D(Q.1) + 30 * D(Q.2) + 25 * D(Q.3) + 25 * D(Q.4)$$
$$= 20 * 0.9424 + 30 * 0.7050 + 25 * 0.8150 + 25 * 0.2713$$
$$= 18.848 + 21.15 + 20.375 + 6.7825$$
$$= 67.155$$
$\cong$ 67 (assuming that no half mark is giving in the total score) (20)

In the following, we generalize the above evaluation method to propose a weighted method for students' answerscripts evaluation using fuzzy sets. Consider a candidate's answerscript to a paper of 100 marks.

Step1: Assume that in total there are n questions to be answered:

TOTAL MARKS = 100
Q.1 carries s_1 marks
Q.2 carries s_2 marks
⋮
Q.n carries s_n marks

Assume that an evaluator evaluates the questions of students' answerscripts using the following four criteria [2]:

C1: Accuracy of Information
C2: Adequate Converage
C3: Conciseness
C4: Clear Expression,

and assume that the weights of the criteria C1, C2, C3, and C4 are w_1, w_2, w_3, and w_4, respectively, where $w_i \in [0, 1]$ and $1 \le i \le 4$. Furthermore, assume that the evaluator can evaluate each question of the students' answerscripts using the above four criteria based on the method described previously. In this case, an evaluator can evaluate the students' answerscripts using a generalized extended fuzzy grade sheet as shown in Table 8, where the degrees of satisfication of the Q.i of a student's answerscript regarding to the criteria C1, C2, C3, and C4 evaluated by the method described previously are D(Ci1), D(Ci2), D(Ci3), and D(Ci4), respectively, where $0 \le D(Ci1) \le 1$, $0 \le D(Ci2) \le 1$, $0 \le D(Ci3) \le 1$, $0 \le D(Ci4) \le 1$, and $1 \le i \le n$.

TABLE 8
A Generalized Extended Fuzzy Grade Sheet

Question Number	Criteria	Satisfication Levels											Degree of Satisfication for Criteria	Degree of Satisfication for criteria
		EG	VVG	VG	G	MG	F	MB	B	VB	VVB	EB		
Q.1	C1												D(C11)	P(Q.1)
	C2												D(C12)	
	C3												D(C13)	
	C4												D(C14)	
Q.2	C1												D(C21)	P(Q.2)
	C2												D(C22)	
	C3												D(C23)	
	C4												D(C24)	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Q.n	C1												D(Cn1)	P(Q.n)
	C2												D(Cn2)	
	C3												D(Cn3)	
	C4												D(Cn4)	

Total Mark = $s_1 * P(Q.1) + s_2 * P(Q.2) + \ldots + s_n * P(Q.n)$

Step 2: The degree of satisfication P(Q.i) of the question Q.i of the student's answerscript can by evaluated as follows:

$$P(Q.i) = \frac{w_1*D(Ci1) + w_2*D(Ci2) + w_3*D(Ci3) + w_4*D(Ci4)}{w_1 + w_2 + w_3 + w_4}, \quad (21)$$

where $P(Q.i) \in [0, 1]$ and $1 \le i \le n$. The total score of the student can be evaluated and is equal to

$$s_1 * P(Q.1) + s_2 * P(Q.2) + \ldots + s_n * P(Q.n) \quad (22)$$

Put this total score in the appropriate box at the bottom of the extended fuzzy grade sheet.

5. Conclusions

In this paper, we have extended the work of [2] to present two new methods for students' answerscripts evaluation. The proposed methods can overcome the drawbacks of the ones presented in [2]. The proposed methods can be executed much faster than the ones presented in [2] due to the fact that they don't need to perform the complicated matching operations. Furthermore, they can make a more fair evaluation of students' answerscript.

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