最佳化布林濾波器設計及其在影像處理方面的應用
Design of Optimal Boolean Filter and Its Applications to Image Processing

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摘要

本文討論最佳化布林濾波器設計的方法以及其在影像處理方面的各種應用。

關鍵字：布林濾波器，堆疊濾波器

Abstract

In this paper, the problem of designing optimal Boolean filter is solved completely. The applications of the optimal Boolean filter have been successively extended to noise suppression, document image enhancement, extraction of text characters from overlapping text/background image, and the edge detection of noisy image corrupted with impulsive noise.

Key words: Boolean filter, stack filter

1. Introduction

The performance of nonlinear filter in image processing has been extensively studied for decades. However, the design of optimal nonlinear filter has not been well defined until the problem of optimal stack filter initialized by Coyle etc. Coyle [1] introduced the class of stack filters and found the connection between the stack filter and positive Boolean function. In [2], Coyle and Lin defined the optimal stack filter under the mean absolute error criterion and the problem of designing optimal stack filter was transformed to that of finding optimal positive Boolean function. Since then many different approaches have been proposed in the finding of optimal stack filter [3][4][5][6]. Most of them belong to the method of training approach since the problem of finding optimal stack filter is considered as a Linear Programming problem. In [1] Coyle also defined the class of weighted order statistic (WOS) filters based on positive threshold function. The WOS filter is a subclass of stack filter and can be represented by a weight vector and a threshold value. The positive threshold functions possess the stacking property, threshold decomposition property, and linear separability. The optimal design and properties of WOS filter have also been studied by [7] and [8].

An extension of stack filter called Boolean filter was introduced by Lee[9]. In the paper, the multilevel representation and several properties of the Boolean filter have been studied . A significant property about the Boolean filter is the decomposition of Boolean filter into a linear combination of stack filters. This property implies that the Boolean filter can be realized by parallel stack filters and thus improves the practical usefulness of Boolean filter.

In this paper, we will concentrate on the design and applications of Boolean filter. The optimal problem of Boolean filter is defined under the MAE criterion. The proposed algorithm can find the optimal Boolean filter in a very simple way. In our approach, the statistical measurement between the observed image and desired image is the dominated computational task. The MAE of Boolean filter is represented in terms of the error incurred by the input vectors. In this way, the optimal Boolean filter can be obtained immediately. Though, a similar approach has been proposed in a recent paper [10] in the finding of optimal Boolean filter. In this paper, we have derived a more precise equation which represents the MAE of the Boolean filter in terms of the error incurred by the input vectors.

The rest of this paper is organized as follows. In section 2, we will briefly review the definition of Boolean filter and the representation of Boolean function. In section 3, the problem of optimal Boolean filter is studied and the relationship of the MAE of a Boolean filter and the cost function of input vectors is derived. Section 4 discusses the decomposition of Boolean filter into a linear combination of stack filters. The decomposition procedure is illustrated concisely with the help of Hasse diagram. The applications of Boolean filter in the area of image processing including impulsive noise suppression, document image enhancement, test extraction from overlapping text/background image and edge detection in impulsive noise environment will be presented in section 5. Real images are tested to evaluate the performance of the Boolean filter. Finally, conclusions are given in section 6.

2. Boolean Filter

The Boolean filter is defined on Boolean function possessing the threshold decomposition property. Let $BF_f(.)$ denote a Boolean filter specified by Boolean function $f(\cdot)$, X be the input gray scale image, T be the thresholding function, and $T_k(X)$ be the thresholded binary image of X thresholded at gray level k. The threshold decomposition property of Boolean filter can be expressed as:

$$BF_f(X) = \sum_{k=1}^{M} f(T_k(X))$$

Owing to the threshold decomposition property, the design, analysis and realization of Boolean filter can then be reduced to binary domain.

In our work, the geometrical representation of Boolean function is adopted. According to the True and False entries of truth table, the 2^n input vectors of a n-variable Boolean function can be classified into two subclasses, one contains the input vectors corresponding to the True entries of truth table which is called on-set, another is called off-set which contains the input vectors corresponding to the False entries of truth table. The Boolean function can be completely specified by the on-set [11] and so does the Boolean filter.

3. Optimal Boolean Filter Under MAE Criterion

In this section, the problem of optimal Boolean filter is revisited and the mean absolute error(MAE) is adopted as the error criterion in determining the optimal Boolean filter. The MAE of a Boolean filter can then be represented in terms of the total error incurred by the input vectors of the on-set. In consequence, the optimal Boolean filter can be found immediately.

For a Boolean filter BF_f where f is the Boolean function defining the Boolean algebra of the Boolean filter, the mean absolute error(MAE) of the Boolean filter is defined as the mean absolute error between the desired image Z and the output of Boolean filter with the observed image X serving as the input.

$$MAE(BF_f) = E[|Z - BF_f(X)|] \qquad (1)$$

where $E[\cdot]$ is the expectation operator.

The optimization problem can be stated as the finding of a Boolean filter which minimizes Equation (1). According to the threshold decomposition property, the MAE of a Boolean filter can be reduced to the sum of the decision errors made by the Boolean filters on each level of thresholded binary images. That is,

$$MAE(S_f) = E[|\sum_{k=1}^{M} T_k(Z) - \sum_{k=1}^{M} f(T_k(X))|] = \sum_{k=1}^{M} E[|T_k(Z) - f(T_k(X))|]$$

The MAE can be further represented as the decision errors incurred by the input vectors. Now, let us define a cost function, $cost(\underline{x})$, as the decision error incurred by $f(\underline{x})$ for deciding a 1 when seeing input vector x.

$$cost(\underline{x}) = C(0, \underline{x}) - C(1, \underline{x})$$

where $C(0, \underline{x})$ is estimated by the number of occurrences of window vector $\underline{x}$ which appears in the observed image when the desired output of this vector is 0, likewise, $C(1, \underline{x})$ is estimated by the number of occurrences of window vector $\underline{x}$ which appears in the observed image when the desired output of this vector is 1. If the window vector $\underline{x}$ is in the on-set of the Boolean filter, then $C(0, \underline{x})$ represents the total decision error of the filter made by $\underline{x}$. Similarly, if the window vector $\underline{x}$ is in the off-set of the Boolean filter, then $C(1, \underline{x})$ represents the total decision error of the filter made by $\underline{x}$. By definition, the MAE of a Boolean filter can be expressed as:

$$\begin{aligned} MAE(BF_f) &= E\{\sum_{k=1}^{M} [\sum_{\underline{x}\in on(f)} C(0, \underline{x}) + \sum_{\underline{x}\in off(f)} C(1, \underline{x})]\} \\ &= E\{\sum_{k=1}^{M} [\sum_{\underline{x}\in on(f)} C(0, \underline{x}) + \sum_{\underline{x}\in off(f)} C(1, \underline{x}) + \sum_{\underline{x}\in on(f)} C(1, \underline{x}) - \sum_{\underline{x}\in on(f)} C(1, \underline{x})]\} \\ &= E\{\sum_{k=1}^{M} \{\sum_{\forall \underline{x}} C(1, \underline{x}) + \sum_{\underline{x}\in on(f)} [C(0, \underline{x}) - C(1, \underline{x})]\}\} \\ &= E\{\sum_{k=1}^{M} [\sum_{\forall \underline{x}} C(1, \underline{x}) + \sum_{\underline{x}\in on(f)} cost(\underline{x})]\} \end{aligned} \qquad (2)$$

where $\sum_{\forall \underline{x}} C(1, \underline{x})$ is a constant for a constant window size. Hence, the MAE of a Boolean filter is equal to the summation of a constant and the total cost of all input vectors in the on-set of the defining Boolean function. It is evident, the optimal Boolean filter that minimizes Equation (2) is specified by the Boolean function whose on-set is composed of all the negative cost input vectors. Hence, the optimal Boolean filter can be obtained immediately after the computation of cost function based on Equation (2).

4. Decomposition of Boolean Filter

Lee [12] has shown that any Boolean filter can be expressed as a linear combination of stack filters. In this section, we will review this property because it is an important property of Boolean filter and the decomposition procedure can be illustrated concisely with the Boolean function being represented by Hasse diagram [13].

The decomposition procedure developed by Lee [13] needs to obtain the *stacking set* and *complement set* of a Boolean function. The stacking set and complement set can be shown explicitly under the representation of Boolean function by the Hasse diagram. For example, for a Boolean function $f = x_1\overline{x_2} + \overline{x_2}x_3 + x_1x_3$, the corresponding Hasse diagram can be depicted as shown in Fig.1. The on-set is *{(100), (001), (101), (111)}*. The stacking set can be obtained immediately by applying the stacking constraint on the on-set which is *{(1,0,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)}*. The complement set is the difference between the stacking set and the on-set which is *{(110), (011)}*.

Actually, the decomposition procedure is to decompose the on-set of the Boolean function into a linear combinations of on-sets of positive Boolean functions. The detail decomposition procedure for this example can be shown as follows:

$$\begin{aligned} on(f) &= \{(1,0,0), (0,0,1), (1,0,1), (1,1,1)\} \\ &= \{(1,0,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)\} \\ &\quad - \{(0,1,1), (1,1,0), (1,1,1)\} + \{(1,1,1)\} \\ &= (x_1 + x_3) - (x_1x_2 + x_2x_3) + (x_1x_2x_3) \end{aligned}$$

Thanks to the property of decomposition, the Boolean filter can then be realized in terms of parallel stack filters. This greatly improves the practical usefulness of Boolean filter.

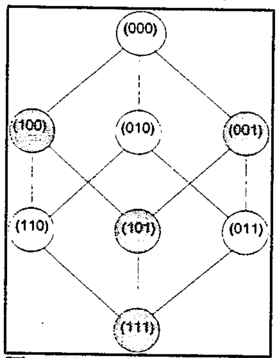

Fig.1. The Hasse diagram of the Boolean function $f = x_1\bar{x}_2 + \bar{x}_2x_3 + x_1x_3$ where the black vertices are the input vectors in the on-set.

5. Applications and Experimental Results

In this section, we will illustrate the applications of optimal Boolean filter in image processing, which include the suppression of image impulsive noise, edge detection, and document image enhancement.

(1)Suppression of image impulsive noise

The excellent performance of median filter and stack filter in the suppression of impulsive noise has been studied extensively. As we know, Boolean filter is a generalized filter with median filter and stack filter as special cases. Consequently, it is doubtless that the optimal Boolean filter will also have good performance at the suppression of impulsive noise.

In this experiment, the testing image is sampled at 256×256 and its corrupted version is generated by adding 45% impulse gray scale noise. The window size of Boolean filter is 3×3. The capability of optimal Boolean filter in the suppression of impulsive noise can be demonstrated by Fig. 2, where Figs. 2(a) and 2(c) are the corrupted images and Figs. 2(b) and 2(d) are the output images generated by optimal Boolean filters.

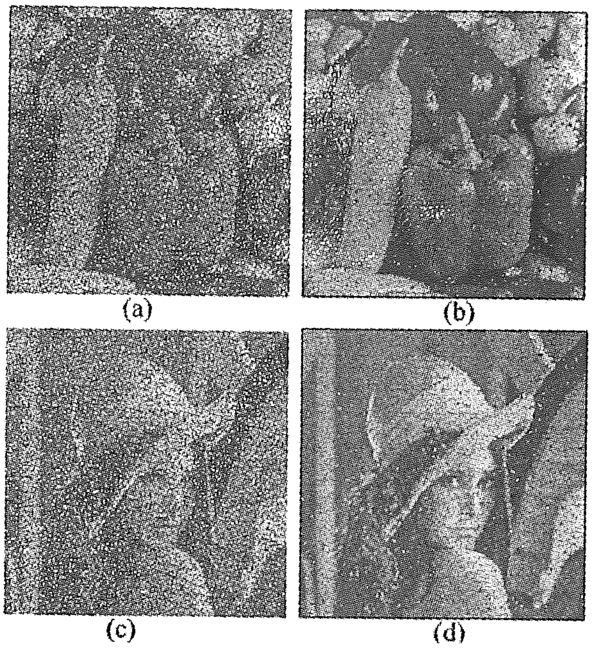

Fig. 2. The performance of optimal Boolean filter in the suppression of impulsive noise. (a) the corrupted "*Pepper*" by adding 45% impulsive noise, MAE = 27.435 ; (b) the enhanced image generated by 3×3 optimal Boolean filter, MAE = 7.061; (c) the corrupted "*Lena*" by adding 45% impulsive noise, MAE = 25.668 ; (b) the enhanced image generated by 3×3 optimal Boolean filter, MAE = 6.362.

(2) Document image enhancement

In this subsection, the optimal Boolean filter is applied to enhance document images. Suppose that we get a noisy document image corrupted by impulsive noise during transmission. In this case, the noise cannot be eliminated by thresholding because the gray level of noise is the same as the text characters. Experimental results show that the optimal Boolean filter is efficient in eliminating the impulsive noise of document image. Let us compare the performance of optimal Boolean filter with that of median filter and a heuristic algorithm which enhances the document image by deleting the isolated points. Fig. 3(a) is the 256×256 testing noisy document image corrupted by 10% impulsive noise, Fig. 3(b) is the enhanced image generated by 3×3 median filter, Fig. 3(c) is the enhanced image by deleting isolated points and Fig. 3(d) is the enhanced image generated by 3×3 optimal Boolean filter. The results show that the optimal Boolean filter is the best one. In addition, it is interesting to note that the median filter has very good performance in eliminating the impulsive noise of gray scaled image. However, it does not work well for the impulsive noise in document image. The median filter eliminates not only the noise but also the text characters as is shown in Fig. 3(b).

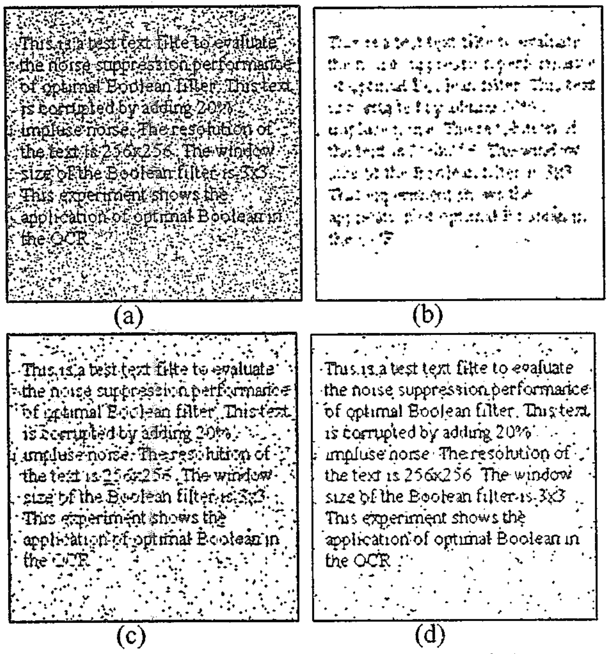

Fig. 3. Comparison of the performance of document image enhancement between different approaches. (a) the testing noisy document image corrupted by adding 10% impulsive noise; (b) the enhanced image generated by 3×3 median filter; (c) the result image by deleting isolated points; (d) the enhanced image generated by 3×3 optimal Boolean filter.

(3) Text extraction from overlapping text/background image

Occasionally, texts will be printed over uniformly distributed graphical background to beautify the layout of articles or attract the readers. Though it is not difficult for humans to read these characters, the extraction of the characters from overlapping background image is still a prerequisite to facilitate the process of Optical Character Recognition (OCR). Morphological approach [14] is effective for this subject to deal with this kind of problems. However, it is time consuming. We found optimal Boolean filter is effective in separating graphic background and text characters when the background is composed of uniformly dots as the image shown in Fig. 4(a). The optimal Boolean filter can extract the text characters completely as shown in Fig. 4(b).

Comparing to the approach of morphology, the advantages of the optimal Boolean filter approach are:

1. The design method of optimal Boolean filter is simpler and the extraction process is faster.
2. The extraction of text characters outperforms the approach of morphology. Take Fig. 4(a) as an example, the result generated by morphological approach contains more noise on the characters as shown in Fig. 5.

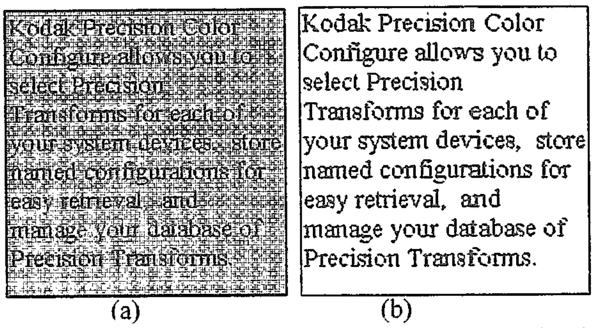

Fig. 4. The result of optimal Boolean filter in the extraction of text characters from overlapped text/background image. (a) a testing document image with overlapped text/background; (b) the output image generated by the optimal Boolean filter.

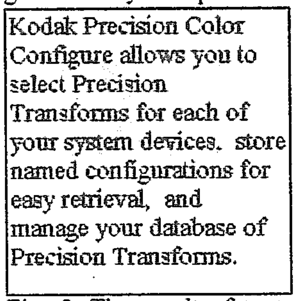

Fig. 5. The result of text extraction by morphological approach for Fig. 4(a).

(4)Edge detection in noisy environment

In designing the optimal Boolean filter, the cost parameters are computed from the image and its corrupted version which are used to determine the on-set of optimal Boolean filter. These statistical estimations reveal the structural characteristics of the image and hence can be used to restore the image from its corrupted version. However, in edge detection application, the cost parameters are computed from the original image and its edge version. This kind of optimal Boolean filter takes the edge image as the referencing image is called optimal Boolean edge detection filter. Experimental results show that this approach is robust to detect the edge in noisy environment.

The referencing edge image used to find the optimal Boolean edge detection filter is generated by the Sobel operator [15]. Fig. 6(b) is the result image generated by 3×3 optimal Boolean edge detection filter with the Fig. 6(a) as the input. Note that there still have noise existing in the image of Fig. 6(b). When carefully examining these noise, we found that they are impulsive type noise and can be eliminated by deleting some input vectors from the optimal Boolean edge detection filter. Since, in binary image, suppose the gray levels are 1 and 0, then, the impulsive noise will be looked as an isolated 1 or isolated 0, such as *(010)* or *(101)*. Whereas the Boolean filter is specified by the input vectors, hence, we can eliminate the impulsive noise by deleting the impulsive type input vectors. For window size of 3×3, the input vectors, *(000001001)* and *(111110110)* are two examples of impulsive type input vectors. The optimal Boolean edge detection filter after the deletion of impulsive type input vectors is called reduced optimal Boolean edge detection filter.

The result image generated by the reduced optimal Boolean edge detection filter is shown in Fig. 6(c). Besides, the reduced optimal Boolean edge detection filter can also be applied to another noisy image and obtain an acceptable result as shown in Fig. 7. The trained reduced optimal Boolean edge detection filter has been obtained from other image and has similar performance. Hence the training optimal Boolean filter is also feasible in edge detection application.

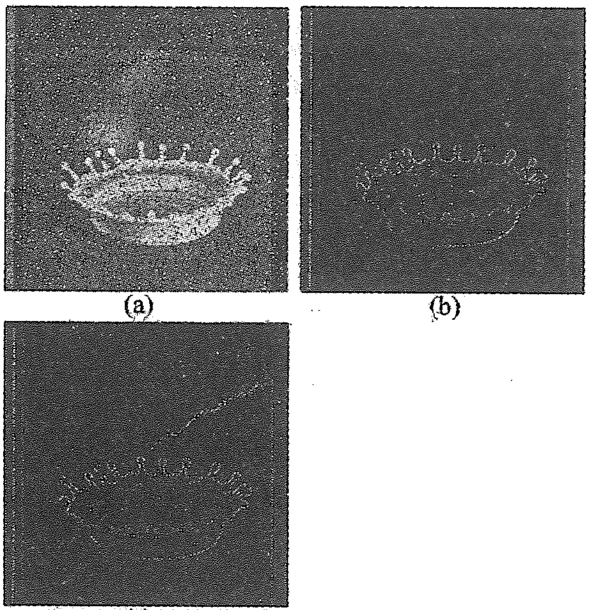

Fig. 6. The result of reduced optimal Boolean edge detection filter in edge detection. (a) the corrupted "*Milk*"with 10% impulsive noise; (b) the output image generated by 3×3 optimal Boolean edge detection filter with the corrupted "*Milk*" as the input; (c) the output image generated by reduced optimal Boolean edge detection filter.

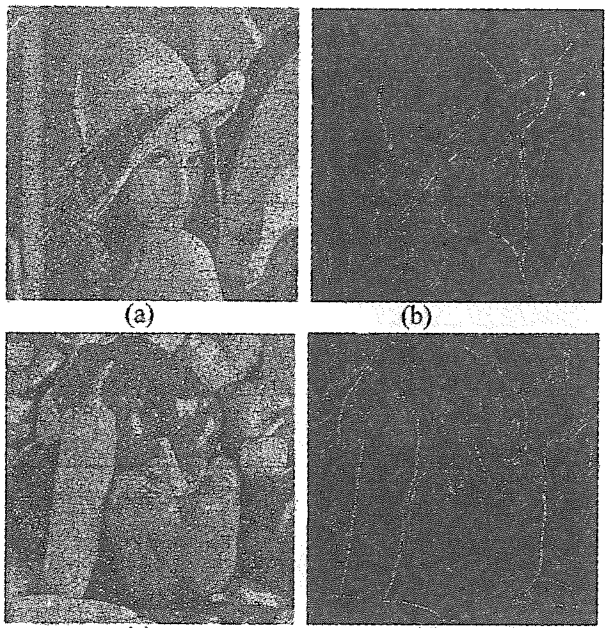

Fig. 7. The result of training optimal Boolean filter in edge detection. (a) and (b) are the testing images; (c) and (d) are the corresponding result images generated by training Boolean filter.

6. Conclusions

In this paper, we discuss the class of Boolean filter which contains stack filter and WOS filter. Hence, it is nature that the Boolean filter will own much more potential in image processing than that of stack filter and WOS filter. Besides, the Boolean filter can be realized in a parallel architecture manner, due to the property of decomposition into a linear combination of stack filters. This improve the practical usefulness of Boolean filter. Furthermor, the finding of optimal Boolean filter is simple.

The good performance of Boolean filter shown in this paper confirms the potential of Boolean filter in image processing. One observation from experiments reveals the fact that the Boolean filter is effective to suppress the impulsive signal no matter that the impulsive signal is the noise being suppressed or is the background being removed. Especially, the experimental result of text characters extraction from overlapping text/background image reminds us that Boolean filter may be suitable for the image segmentation and texture analysis, which becomes our further work in the study of Boolean filter.

Reference

[1] P. D. Wendt, E. J. Coyle, and N. C. Gallagher, "Stack filters", *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, No. 4, pp. 898-911, August 1986.

[2] E. J. Coyle, and J.-H. Lin, "Stack filters and the mean absolute error criterion," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 36, pp. 1244-1254,

Aug. 1988.

[3] M. Gabbouj, and E. J. Coyle, "On the LP which finds a MMAE stack filter," *IEEE Trans. Signal Processing*, vol. 39. no. 11, pp. 2419-2424, Nov. 1991.

[4]J. -H. Lin, and E. J. Coyle, "Minimum mean absolute error estimation over the class of generalized stack filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, no. 4, pp. 663-678. Apr. 1990.

[5] J.-H. Lin, T. M. Sellke, E. J. Coyle, " Adaptive stack filtering under the mean absolute error criterion", *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38,, pp. 938-54. Apr. 1990.

[6] B. Zeng, M. Gabbouj and Y. Neuvo "A unified design method for rank order, stack, and generalized stack filters based on classical Bayes decision," *IEEE Trans. Circuits and Systems*, vol. 38, pp. 1003-20, 1991.

[7] L. Yin, J. Astola, Y. Neuvo, "Optimal weighted order statistic filters under the mean absolute error criterion", *1991 International conference on Acoustics, Speech and Signal Processing*, pp. 2529-32, 1991.

[8] P. -T. Yu, W. -H. Liao, "Weighted order statistics filters - their classification, some properties, and conversion algorithm", *IEEE Trans. Signal Processing*, vol. 42. no. 10, pp. 2678-91, Aug. 1994.

[9] K. D. Lee, Y. H. Lee, "Threshold Boolean filter", *IEEE Trans. Signal Processing*, vol. 42. no. 8, pp. 2022-36, Aug. 1994.

[10] I. Tabus, D. Petrescu, M. Gabbouj, "A training framework for stack and Boolean filtering — fast optimal design procedures and robustness", *IEEE Trans. Signal Processing*, vol. 5. no. 6, pp. 809-825, June 1996.

[11] S. Muroga, *Threshold Logic and Its Applications*. New York: Wiley, 1971, p. 18.

[12] K. D. Lee and Y. H. Lee, " Threshold Boolean filters", *IEEE Trans. Signal Processing*, vol. 42, pp. 2022-2036, Aug. 1994.

[13]P. -T. Yu, "Some representation properties of stack filters," *IEEE Trans. Signal Processing*, vol. 40. no. 9, pp. 2261-2266, Sept. 1992.

[14] S. Liang, M. Ahmadi, M. Shridhar, " A morphological approach to text string extraction from regular periodic overlapping text/background images", *CVGIP: Graphical Models and Image Processing*, Vol. 56, No. 5, pp. 402-413, 1994

[15] R. C. Gonzales, R. E. Woods, *Digital Image Processing*, Addison Wessley, 1993