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一個新的模糊列等方法解最佳化問題
A New Fuzzy Ranking Approach to Optimization Problems

孫光天
Sun Koun-Tem
台南師院資訊教育研究所
Institute of Information Education
National Tainan Teachers College
ktsun@ipx.ntntc.edu.tw

王俊義
Wang Chun-I
台南師院資訊教育研究所
Institute of Information Education
National Tainan Teachers College
m85513@ipx.ntntc.edu.tw

黃志裕
Huang Chih-Yue
台南師院資訊教育研究所
Institute of Information Education
National Tainan Teachers College
m85512@ipx.ntntc.edu.tw

摘要

在本文中，我們提出一個新的模糊列等方法來解最佳化問題，並選用了一般化且截止日允許延遲的工作排序問題為例，進行模擬，結果顯示優於傳統的貪婪演算法和其他模糊列等方法。

關鍵字: 多維模糊列等法,中心點, 最佳化問題

Abstract

In this paper, we introduce a new fuzzy ranking method to solve optimization problems. The generalized and deferred allowable job sequencing with deadline problem is used as an example to simulate and our results are better than the greedy approach and other fuzzy ranking methods.

Keywords: multi-dimensional fuzzy ranking method, centroid point, optimization problems

1. Introduction

Most of optimization problems are the kind of NP-complete problems [1,2], and currently no polynominal time algorithm exists for finding the optimal solution. Some heuristic methods such as branch-and-bound [3] and prune-and-search [4] have been proposed to solve problems of this kind. However, the time complexity increases exponentially for these traditional methods and becomes excessively for large problems. Recently, fuzzy set theories[5] and neural networks[6,7] have been used to solve these problems, and obtain good solutions. Although fuzzy set concepts are mainly used in linguistic domains such as uncertainty[8,9,10], they can be also used in numerical domains, where each number is assigned to a membership value. The ranking methods of fuzzy set theory have been also applied to decision and problems[11,12,13]. For these problems, the fuzzy ranking number is used to determine the importance or sequencing of fuzzy sets (objects). Based on the ranking number of each object, a good solution can be obtained. However, for job sequencing with deadline problems[14], there is still no good fuzzy ranking method can be applied to solve this problem efficiently. In this paper, a new ranking method is proposed which is something like determining the center of a graph. The resulting centroid point is then used to order the sequence of jobs, and thus the total profit (or penalty) is maximized (or minimized). In the next section, we will discuss in detail job sequencing with deadline problems, and some additional conditions for deferred processing will be also explained. Section 3 will briefly review some related fuzzy ranking methods. Section 4 will introduce our ranking method and illustrate its merit. Section 5 will show simulation results which are better than other approaches. Finally, we present a concluding summary of our research and make some suggestions for future work.

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2. Review of Generalized Job Sequencing with Deadline Problems

Job sequencing with deadline problems can be defined as: given a set of n jobs, with each job i having an integer deadline $d_i \geqq 0$ and a profit $P_i \geqq 0$. For any job i, the profit P_i is earned if this job is completed by its deadline. In addition, each job is completed by a machine in one unit of time, and only one machine is available for each job. A feasible solution is a subset, J, of jobs such that each job in this subset can be completed by its deadline, and the value of a feasible solution J is the sum of profits of jobs in J. An optimal solution is a feasible solution with the maximum value. For example, suppose there are four jobs (J_1, J_2, J_3, J_4) with profits (P_1, P_2, P_3, P_4) = (20, 40, 60, 80) and deadlines (d_1, d_2, d_3, d_4) = (1, 2, 2, 2). We can obtain the following feasible solutions and their corresponding profits: [(J_i, J_j, J_k, …), profit] = [(J_1, J_2), 60], [(J_1, J_3), 80], [(J_1, J_4), 100], [(J_2, J_3), 100], [(J_2, J_4), 120], [(J_3, J_4), 140], and the solution [(J_3, J_4), 140] is also an optimal solution. This optimal solution can be obtained by a greedy algorithm which sorts each job's profit and selects them in order. However, if each job's processing time is not an unit time, then the generalized problem become very difficult to solve and it is known as an NP-complete problem. Each job J_i has associated with it a three tuple (P_i, d_i, t_i), where P_i is the value of profit, d_i is the deadline, and t_i is the required processing time. Figure 1 illustrates an example of a generalized job sequencing with deadline problem. In Figure 1, the integer i of $[i]_j$ denotes the units of required processing time of job j. The optimal solution is {J_4, J_1, J_3}, and profit 100 (=20 + 50 +30) is maximum.

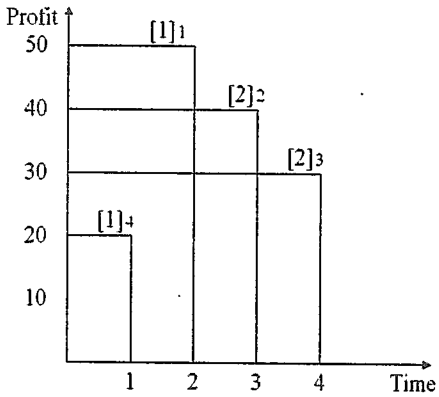

Figure 1. A graph representation of a generalized job sequencing with deadline problem; there are 4 jobs: 1, 2, 3, and 4 with deadlines 2, 3, 4, and 1, respectively.

In real situations, when a job is completed beyond the deadline, the profit does not become zero instantly, but rather we pay an additional cost for this delay and then obtain less profit. Figure 2 illustrates the real situation of a generalized job sequencing with deadline problem.

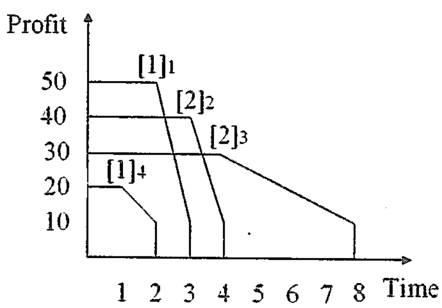

Figure 2. A generalized and deferred allowable job sequencing with deadline problem.

In this situation, the difficulty of this problem has been increased, and then this problem is still an NP-complete problem. The solution of Figure 2 becomes {J_1, J_2, J_3} with a profit 115 (=50 + 40 + 25), which is different from that of Figure 1, and the profit of Figure 2 is better than Figure 1. Even if the processing time of each job in Figure 2 is set to one unit of time, it is still an NP-complete problem. The traditional greedy approach can only find a good solution but not an optimal solution. Section 5 will show the simulation results of the greedy approach.

3. Fuzzy Ranking Methods

Many fuzzy ranking methods have been proposed for solving decision and optimization problems such that a good solution can be obtained [12,13]. Yager's ranking method [15], the "weighted average ranking function," is one of the most widely used methods for optimization problems. The ranking function $F(A_i)$ is defined:

$$F(A_i) = \frac{\int g(x)\, \mu_{A_i}(x)\, dx}{\int \mu_{A_i}(x)\, dx}, \text{ for continuous values of x,} \quad (1)$$

$$\text{or } F(A_i) = \frac{\sum g(x) \cdot \mu_{A_i}(x)}{\sum \mu_{A_i}(x)}, \text{ for discrete values of x,} \quad (2)$$

where the weight g(x) is the measure of importance. If g(x) is assumed as a linear weight, that is $g(x) \equiv x$, then $F(A_i)$ represents the centroid of the fuzzy set A_i. Yager's ranking method is also often used in fuzzy inference for defuzzification [16-19]. Figures 3 (a) and (b) illustrate an example of the weighted average method with two different types of fuzzy functions (triangle and trapezoid).

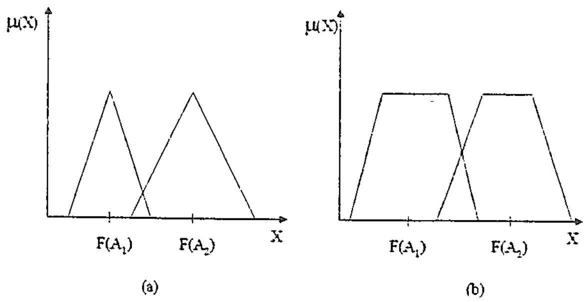

Figure 3. The weighted average method for (a) triangular fuzzy sets and (b) trapezoidal fuzzy sets.

When $F(A_1) < F(A_2)$, it implies $A_1 < A_2$. If the importance of each fuzzy set is not only dependent on the value of X but also depends on the value of membership function μ (X)(=y), then Yager's method can be extended into a two-dimension value Y(x,y) for the fuzzy set A_i.

$$Y(x,y):\begin{cases} x = \dfrac{\sum x\cdot\mu(x)}{\sum \mu(x)},\ \forall\,\mu(x)>0 \text{ and } x\in A_i. & (3)\\[2ex] y = \dfrac{\sum x\cdot\mu(x)}{\sum x},\ \forall\,\mu(x)>0 \text{ and } x\in A_i. & (4)\end{cases}$$

Figures 4 (a) and (b) show the values of Y(x, y) for the same problems in Figure 3.

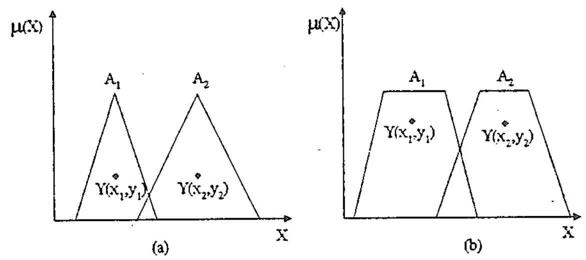

Figure 4. The centroid points $Y(x_1,y_1)$ and $Y(x_2,y_2)$ of the extended Yager's method for the same examples as in Figure 3.

When x_1 and x_2 are considered under one criterion, the importance of a fuzzy set can be defined by the value of x (i.e., if $x_1 > x_2$, then $A_1 > A_2$), and y_1 and y_2 can be also considered for another criterion. However, the centeriod points, Ys, of Yager's method are not near the centers of fuzzy sets. In the next section, to better the results of the extended Yager's method, we will introduce a new ranking method which is similar to finding the center of a graph.

4. The Proposed Ranking Method

The basic idea of our ranking method is to determine a two-dimensional value which approaches the center of the graph of a fuzzy set. For a triangular fuzzy set A_i, the determined value is exactly equal to the center, C, of A_i (as shown in Figure 5).

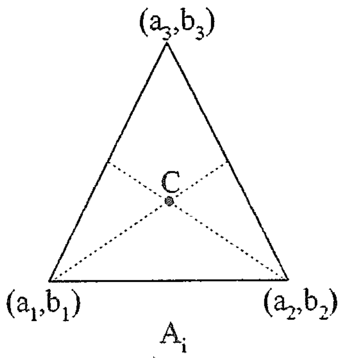

Figure 5. Point C is the center of graph A_i.

When neither (a_1,b_1) nor (a_2,b_2) is (0,0), the values of x and y in point C(x,y) are:

$$x = \frac{\beta_1-\beta_2}{\beta_1\cdot\alpha_2-\beta_2\cdot\alpha_1}, \qquad (5)$$

and

$$y = \frac{\alpha_1-\alpha_2}{\alpha_1\cdot\beta_2-\alpha_2\cdot\beta_1}. \qquad (6)$$

where

$$\alpha_1 = \frac{2b_1-b_2-b_3}{(a_2+a_3)\cdot b_1-(b_2+b_3)\cdot a_1}, \qquad (7)$$

$$\alpha_2 = \frac{2b_2-b_1-b_3}{(a_1+a_3)\cdot b_2-(b_1+b_3)\cdot a_2}, \qquad (8)$$

$$\beta_1 = \frac{(a_2+a_3)-2a_1}{(a_2+a_3)\cdot b_1-(b_2+b_3)\cdot a_1}, \qquad (9)$$

and

$$\beta_2 = \frac{(a_1+a_3)-2a_2}{(a_1+a_3)\cdot b_2-(b_1+b_3)\cdot a_2}, \qquad (10)$$

where, if one of three corners of a triangle is (0,0), then (a_3,b_3) can be assigned to this corner (0,0). For a trapezoidal fuzzy set A_j, the determined centroid point is approximately the center of A_j, and the computation steps are:

(1) Divide the trapezoid A_j into three triangles with a common lower-left corner.
(2) Find three centers, (a_1,b_1), (a_2,b_2) and (a_3,b_3), for these three triangles.
(3) Connect these three points, (a_1,b_1), (a_2,b_2) and (a_3,b_3), into a new triangle T.
(4) Find the center, C(x,y), of T.
(5) The point C(x,y) will be close to the center of the trapezoidal fuzzy set A_j, and it is used as the fuzzy ranking value of A_j.

Figure 6 illustrates an example of our approximation method. The position of point C(x,y) is independent

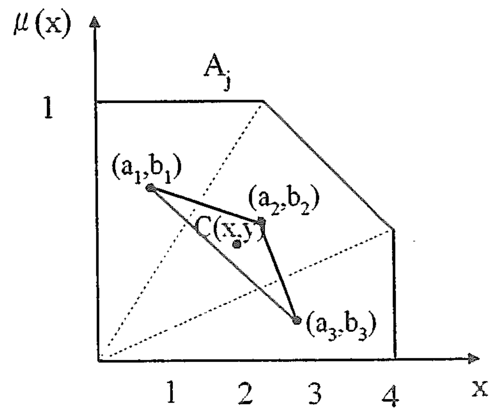

Figure 6. The centroid C(x,y) is the ranking value of the trapezoidal fuzzy set A_j.

of the scale of μ(x), i.e., if the range of μ(x) is changed from [0, 1] to [0, 100], then the new point C'(x',y') is still located in the same position but x' = x and y' = 100 * y. To solve the generalized and deferred allowable job sequencing with deadline problem, we propose an algorithm which selects the jobs by using our fuzzy ranking method.
The steps are:

(1)a: For each trapezoidal fuzzy set A_i, find the fuzzy ranking value, $C_{Ai}(x,y)$, by applying our fuzzy ranking method.

b: For each rectangular fuzzy set A_j, the fuzzy ranking value, $C_{Aj}(x,y)$, is defined as:

$$x = \frac{\sum x_i}{4}, \ \forall x_i \in \text{corners } (x_i, y_i) \text{ of } A_j. \quad (11)$$

$$y = \frac{\sum y_i}{4}, \ \forall y_i \in \text{corners } (x_i, y_i) \text{ of } A_j. \quad (12)$$

(2) Sort all fuzzy ranking values, $C_{Ak}(x,y)$, by the decreasing order of the value x.

(3) Apply the Max-Max function to the fuzzy ranking values, $C_{Ak}(x,y)$, for scheduling jobs, and the value of x is determined by the upper ceiling function

$$C^*_{A_m}(x^*, y^*) = \underset{y}{Max}\underset{\lceil x \rceil}{Max}\{C_{A_k}(x,y), \forall A_k\}. \quad (13)$$

(4) a: If the date at the deadline of job m is unallocated, then allocate this date to job m.

b: If the date at the deadline of job m is allocated by another job, then find an unallocated date backward until the time is equal to x^*, and then assign the job m to the first unallocated date.

c: If (4)a and (4)b cannot be satisfied, then insert the job m into the unallocated list, UL.

d: Go to step (3) until all fuzzy ranking values are considered.

(5) Apply the Max function to the UL.

$$C^*_{A_v}(x^*, y^*) = \underset{y}{Max}\{C_{A_u}(x,y), \forall A_u \in UL\}. \quad (14)$$

(6) a: Find an unallocated date backward from the date x* -1 to 1 and assign the job v to the first unallocated date.

b: If (6)a can not be satisfied, then find an unallocated date forward from the date x* until the date with profit = 0, and assign the job v to the first unallocated date.

c: If there is no date that can be allocated for the job v, then remove the job v from the UL, and select the next job from the UL.

d: Go to step (5) until the UL is empty.

(7) Compute the total profit for the allocated jobs.

Based on this algorithm, we can efficiently solve the generalized and deferred allowable job sequencing with deadline problem. We have also designed an algorithm by applying the extended Yager's method which is basically an embodiment of our algorithm, but differing from fuzzy ranking method, however, in step (1). If you compare our method with the extended Yager's method (see Figure 7), you can note that our method is more reasonable. Simulation results confirm that our method produces better solutions, as will be shown.

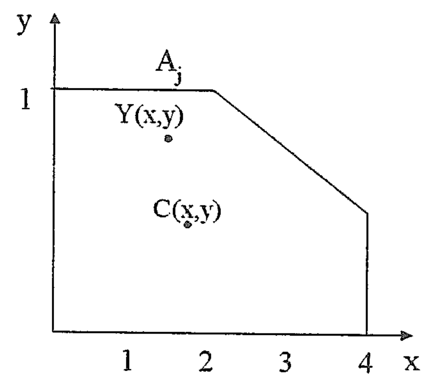

Figure 7. C(x,y) and Y(x,y) are the ranking values of our method and the extended Yager's method, respectively. For fuzzy set A_j , a reasonable importance should be near the center of graph A_j for considering two criteria: x and y.

5. Simulation Results

We randomly selected 100 generalized and deferred allowable job sequencing with deadline problems with the number of jobs being 5, 10, 15, 20, 25, 30, 35, and 40. Figure 8 shows an example of this problem with problem size = 3 (jobs).

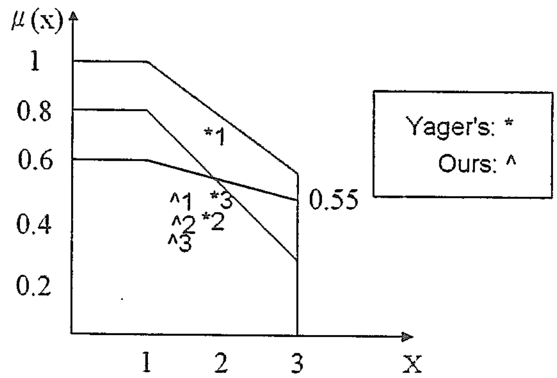

Figure 8. An example of the generalized and deferred allowable job sequencing with deadline problem with problem size = 3. The orders of ranking values are different between ours and the extended Yager's method. The profit of ours is 2.1(=1 + 0.55 + 0.55) which is better than Yager's 1.875 (=1 + 0.575 + 0.3). The rate of improvement of our algorithm reaches 12%.

Simulation results of our method, the extended Yager's method, and the greedy approach are illustrated in Figure 9. Our method obtains better solutions than the other methods, which verifies our approach as useful for solving decision and optimization problems. In addition, we believe that this two-dimensional ranking method can be applied to the solution of complex problems that are required to compare the ranking values under multiple criteria.

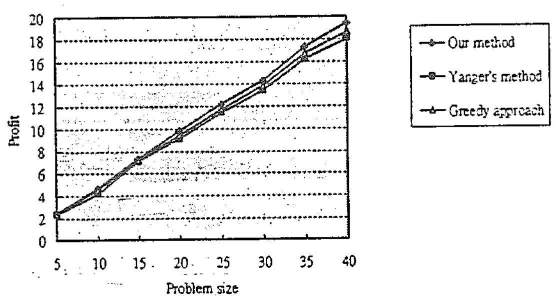

Figure 9. Simulation results of our method, the extended Yager's method, and the greedy approach.

6. Conclusions

Traditional fuzzy ranking methods only generate one real number for each fuzzy set. However, if we consider the objects in a problem under multiple-criteria, then one-dimensional ranking methods do not work well. In this paper, we first propose the concept of a multi-dimensional ranking method to solve traditional optimization problems and then use a two-dimensional ranking method to solve the generalized and deferred allowable job sequencing with deadline problems. The simulation results show that our method obtains better solutions than the traditional greedy algorithm and the extended Yager's fuzzy ranking method. In addition, this multi-dimensional fuzzy ranking method can be extended beyond the two-term example presented here to solve n-term problems. In the future, we will use the proposed fuzzy ranking method to solve other and more complex decision/optimization problems.

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