An Efficient Denoising Method Using Dual-tree Wavelet Transform

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Abstract

Denoising technology is an important operation in image processing. Denoising technologies approximately divides into two types: one is denoising operation in the spatial domain and the other is that in the frequency domain. Denoising operation in frequency domain is better than that the spatial domain in recently researches. The wavelet transform provides a multiresolution representation using a set of analyzing functions that are dilations and translations of a few functions (wavelets). The wavelet transform comes in several forms. The critically-sampled form of the wavelet transform provides the most compact representation, however, it has several limitations to denoise. Therefore, we describe dual-tree complex wavelet transform to solve these limitations. The image edge information maybe loses by denoising, so preserving edge is critical. We adopt the TV filter and Canny filter to preserve and enhance the edge information in low frequency subband. The simulation results demonstrate that that complex dual-tree method removes more noise signal than separable and real methods (all of the methods include both TV filter and Canny filter to preserve the edges). Dual-tree method outperforms separable method.

Keyword: separable wavelet transform**,** Dual-tree complex wavelet transform, TV filter, Canny Filter

I. Introduction

Denoising technology is an important operation in image processing. Actuality, this technology is applied in many kinds of images, such as medical diagnosis image, remote sensor image and Synthetic Aperture Radar (SAR) image. These images have some annoying noises that reduce the image visual quality. Therefore, the researches about denoising develop in the recently [1][2][3]. Denoising technologies approximately divides into two types: one is denoising operation in the spatial domain and the other is that in the frequency domain, for instance, Fourier transform, discrete wavelet transform, and complex dual tree wavelet transform. The latter uses the linear or nonlinear filter to reduce the noise in the frequency domain [4][5]. For many natural signals, the wavelet transform is a more effective tool than the Fourier transform. The wavelet transform provides a multiresolution representation using a set of analyzing functions that are dilations and translations of a few functions (wavelets). The wavelet transform comes in several forms. The critically-sampled form of the wavelet transform provides the most compact representation, however, it has several limitations. For example, it lacks the shift-invariance property, and in multiple dimensions it does a poor job of distinguishing orientations, which is important in image processing. For these reasons, it turns out that for some applications improvements can be obtained by using an expansive wavelet transform in place of a critically-sampled one. In this paper, we describe and provide an implementation of the dual-tree complex discrete wavelet transform.

This paper is organized as follows. In Section II, we describe Brief the research about denoising technology and wavelet transform. The design of our method is described in Section III. Section IV compares the performance of our proposed algorithm with that of other current wavelet-based denoising methods applied on the 'Lena' test image. Finally, Section V draws conclusions and describes future work directions.

II. Background

In this section, we describe the recently denoising technology researches and wavelet transforms.

A. Denoising

Image denoising is an important image processing task, both as a process itself, and as a component in other processes. Very many ways to denoise an image or a set of data exists. The main property of a good image denoising model is that it will remove noise while preserving edges. Traditionally, linear models have been used. One common approach is to use a Gaussian filter, or equivalently solving the heat-equation with the noisy image as input-data. For some purposes this kind of denoising is adequate. One big advantage of linear noise removal models is the speed. But a drawback of the linear models is that they are not able to preserve edges in a good method: edges, which are recognized as discontinuities in the image, are smeared out. On the other hand, nonlinear models can handle edges in a much better way than linear models. One popular model for nonlinear image denoising is the Total Variation (TV) filter, introduced by Rudin, Osher and Fatemi in [6]. This filter is very good at preserving edges, but smootly varying regions in the input image are transformed into piecewise constant regions in the output image. Since smooth regions are transformed into piecewise constant regions when using the TV-filter, research is done to make a model which lets smoothly varying regions be transformed into smoothly varying regions, and still preserves edges. Since total variation minimizing models have become one of the most popular and successful methodology for image restoration.

B. 2D Discrete Wavelet transform

In the 2D case, the 1D analysis filter bank is first applied to the columns of the image and then applied to the rows. If the image has R rows and C columns, then after applying the 1D analysis filter bank to each column we have two subband coefficients, each having R/2 rows and C columns; after applying the 1D analysis filter bank to each row of both of the two subband coefficients, we have four subband coefficients, each having R/2 rows and C/2 columns. This is illustrated in the diagram below. The analysis filter banks of wavelet transform for image processing are show in Fig. 1. The 2D synthesis filter bank combines the four subband coefficients to obtain the original image of size RxC.

As in the 1D case, the 2D discrete wavelet transform of a signal x is implemented by iterating the 2D analysis filter bank on the lowpass subband. In this case, at each scale there are three subbands instead of one.

The perfect reconstruction of the 2D DWT is verified in the following example. We create a random input signal x of size 128 by 64, apply the DWT and its inverse, and show it reconstructs x from the wavelet coefficients. There are three wavelets associated with the 2D wavelet transform. Fig. 2 illustrates three wavelets as gray scale images.

The first two wavelets are oriented in the vertical and horizontal directions; however, the third wavelet does not have a dominant orientation. The third wavelet mixes two diagonal orientations, which gives rise to the checkerboard artifact. (The 2D DWT is poor at isolating the two diagonal orientations.) We describe dual-tree complex wavelet transform to avoid this artifact. The One of the advantages of the dual-tree complex wavelet transform is that it can be used to implement 2D wavelet transforms that are more selective with respect to orientation than the separable 2D DWT.

the details are in next section.

III. Proposed Method

A. Dual-Tree Complex Wavelet Transform

In recently research, for some applications of the discrete wavelet transform, improvements can be obtained by using an expansive wavelet transform in place of a critically-sampled one. There are several kinds of expansive DWTs; here we describe the dual-tree complex discrete wavelet transform.

The dual-tree complex DWT of a signal x is implemented using two critically-sampled DWTs in parallel on the same data, as shown in the Fig. 3.

The transform is 2-times expansive because for an N-point signal it gives 2N DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. However, if the filters are designed is a specific way, then the subband signals of the upper DWT can be interpreted as the real part of a complex wavelet transform, and subband signals of the lower DWT can be interpreted as the imaginary part. Equivalently, for specially designed sets of filters, the wavelet associated with the upper DWT can be an approximate Hilbert transform of the wavelet associated with the lower DWT. When designed in this way, the dual-tree complex DWT is nearly shift-invariant, in contrast with the critically-sampled DWT. Moreover, the dual-tree complex DWT can be used to implement 2D wavelet transforms where each wavelet is oriented, which is especially useful for image processing. The dual-tree complex DWT outperforms the critically-sampled DWT for applications like image denoising and enhancement.

One of the advantages of the dual-tree complex wavelet transform is that it can be used to implement 2D wavelet transforms that are more selective with respect to orientation than is the separable 2D DWT.

There are two versions of the 2D dual-tree wavelet transform: the real 2D dual-tree DWT is 2-times expansive, while the complex 2D dual-tree DWT is 4-times expansive. Both types have wavelets oriented in six distinct directions. We describe the real version first.

The real 2D dual-tree DWT of an image x is implemented using two critically-sampled separable 2D DWTs in parallel. Then for each pair of subbands we take the sum and difference. The six wavelets associated with the real 2D dual-tree DWT are illustrated in Fig. 4 as gray scale images.

Each of the six wavelets is oriented in a distinct direction. Unlike the critically-sampled separable DWT, all of the wavelets are free of checker board artifact. Each subband of the 2D dual-tree transform corresponds to a specific orientation.

The complex 2D dual-tree DWT also gives rise to wavelets in six distinct directions, however, in this case there are two wavelets in each direction as will be illustrated below. In each direction, one of the two wavelets can be interpreted as the real part of a complex-valued 2D wavelet, while the other wavelet can be interpreted as the imaginary part of a complex-valued 2D wavelet. Because the complex version has twice as many wavelets as the real version of the transform, the complex version is 4-times expansive. The complex 2D dual-tree is implemented as four critically-sampled separable 2D DWTs operating in parallel. However, different filter sets are used along the rows and columns. As in the real case, the sum and difference of subband images is performed to obtain the oriented wavelets. The twelve wavelets associated with the real 2D dual-tree DWT are illustrated in the Fig. 5 as gray scale images.

Note that the wavelets are oriented in the same six directions as those of the real 2-D dual-tree DWT. However, here we have two in each direction. If the six wavelets displayed on the first row are interpreted as the real part of a set of six complex wavelets, then the magnitudes of the six complex numbers are shown on the third row. As shown in the figure, the magnitudes of the complex wavelets do not have an oscillatory behavior - instead they are bell-shaped envelopes.

B. Denoiseing Method

We can choose variance transform, such as separable DWT, real Dual-Tree DWT, and complex Dual-Tree DWT to decompose the image, and then use the method to remove the noise from an image. In this section, these methods will be introduced and comparison will also be made.

One technique for denoising is wavelet thresholding in the high frequency subbands. Some of the resulting wavelet coefficients correspond to details in the data set (high frequency subbands). If the details are small, they might be omitted without substantially affecting the main features of the data set. The idea of thresholding is to set all high frequency subband coefficients that are less than a particular threshold to zero. These coefficients are used in an inverse wavelet transformation to reconstruct the data set.

Total Variation (TV) filter is very good at preserving edges. This filter commonly using in spatial domain. We use this filter in low frequency subband to preserve the edges and enhance the edges. The canny filter [7] is simple and fast to detect the edges and then use this information to enhance the edges.

Fig. 6 shows the step of the proposed method. First, proposed method uses dual-tree wavelet transform in input image and then handles the low frequency subband (LL) and high frequency subbands (HL, LH and HH) independently. The TV filter and canny filter are adopted to denoise and enhance the edges in the low frequency subband (LL). And the wavelet thresholding denies in high frequency subbands. All of the denoising functions operate in the frequency domain and then inverse dual-tree wavelet transform to output image.

IV. Simulation results

The performance is evaluated by PSNR (peak signal to noise ratio). PSNR is mathematically evaluated as

$$\mathrm{PSNR} = 10\log_{10} \frac{255^2}{\frac{1}{T}\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\left(x_{i,j} - x'_{i,j}\right)^2}$$

PSNR has been accepted as a widely used measure of quality in the field of image compression.

The test original image is Lena (Fig. 7(a)) that is gray level image with a size of 512 x 512 pixels with 8 bpp. In Fig.7(b), The noise adding in original image is Gaussian white noise that mean sets zero and variance sets 0.01.

Wavelet thresholding is applied to wavelet coefficients through all scales and subbands. Function sets coefficients with values less than the threshold to 0, then substracts T from the non-zero coefficients. We use a threshold value of 40, which is the optimal threshold point for this case. Both TV filter and Canny filter are combined in the low frequency subband. After thresholding and filtering, we take inverse wavelet transform.

The PSNR value of the reconstructive image that decomposed by separable wavelet transform is 27.28 dB and the one that decomposed by dual-tree wavelet transform is 27.45 dB. The PSNR value difference is not obvious, and we show the reconstructive image in Fig. 8. It is apparent that the visual quality of reconstructive image by complex dual-tree DWT (Fig. 8(b)) is better than that by separable DWT (Fig. 8(a)).

But PSNR and the visual quality are not absolutely relative. The PSNR value of the higher visual quality image may be smaller than that of the lower one. The PSNR value of Fig. 9(a) is 27.09 dB and that of Fig. 9(b) is 27.40 dB. But the visual quality of Fig. 9(b) is not better than that of Fig. 9(a). Therefore, we enlarge the figures and focus on a small area of these figures to see the difference. And we demonstrate that the visual quality of complex dual-tree decomposed reconstructive image (Fig. 9(a)) is better than that of real dual-tree decomposed reconstructive image (Fig. 9(b)).

V. Conclusions and Future works

The simulation results demonstrate that complex dual-tree method removes more noise signal than separable and real methods do. Dual-tree method outperforms separable method. It illustrates the denoising capability for three different methods: complex 2-D dual-tree method is the best, followed by real 2-D dual-tree method and separable method. In this method, the threshold (T) is heuristic, so our future work is to predict the threshold by statistics. And the other future work is to reduce the time complexity in complex dual-tree wavelet transform computing.

VI. Acknowledgement

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VII. References

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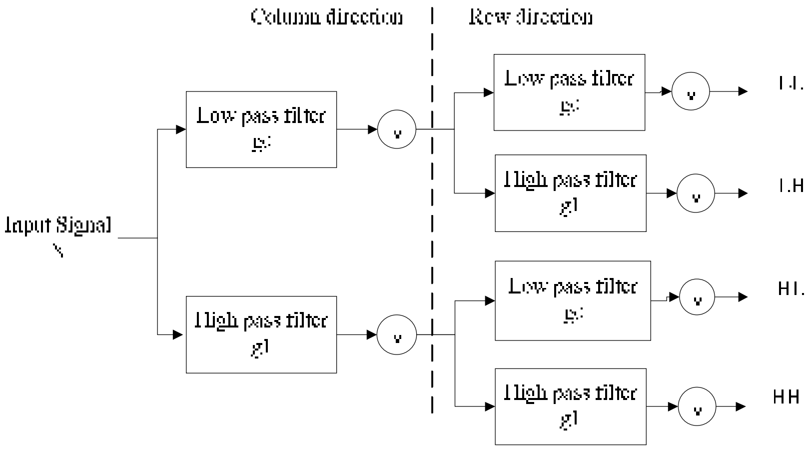

Fig. 1: One stage in multi-resolution wavelet decomposition of an image

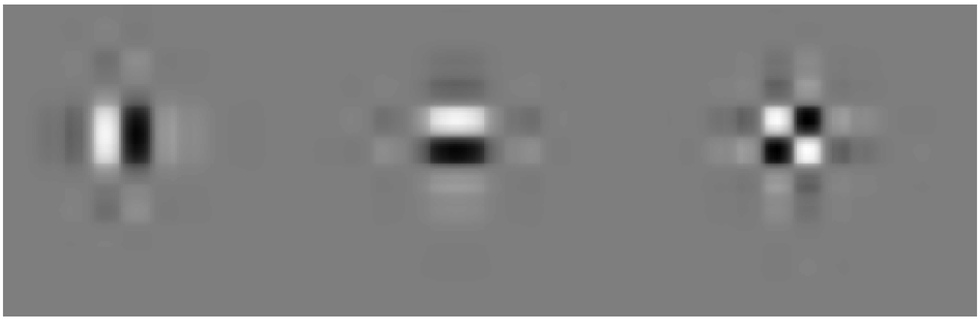

Fig. 2: Three wavelets as gray scale images

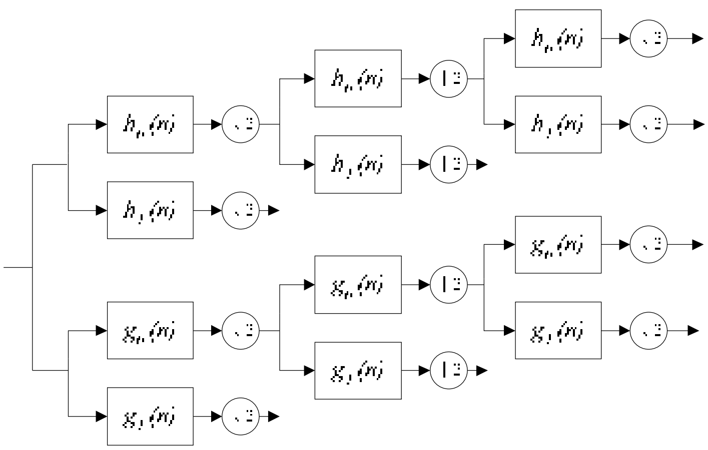

Fig. 3: Decomposition of dual-tree wavelet transform

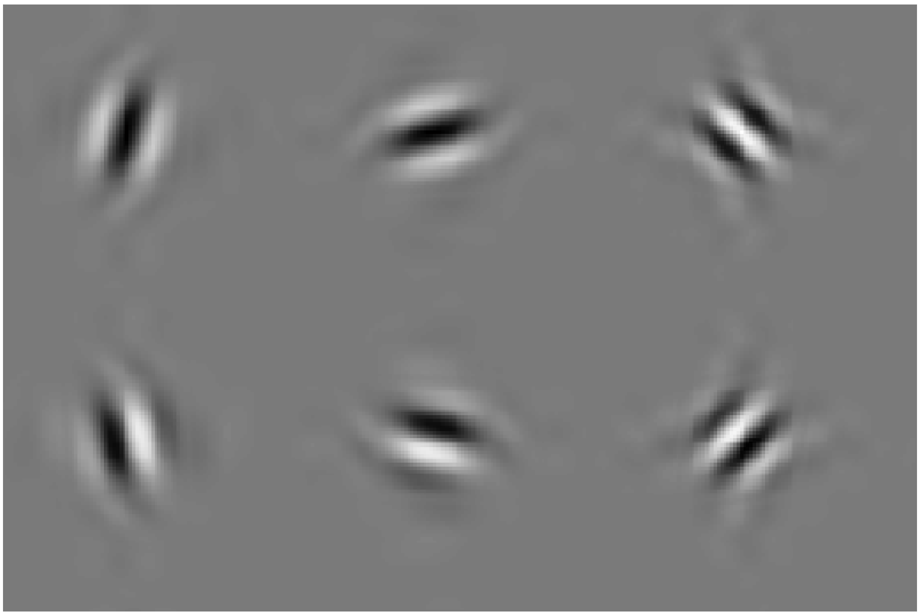

Fig. 4: Directional wavelets for 2D DWT

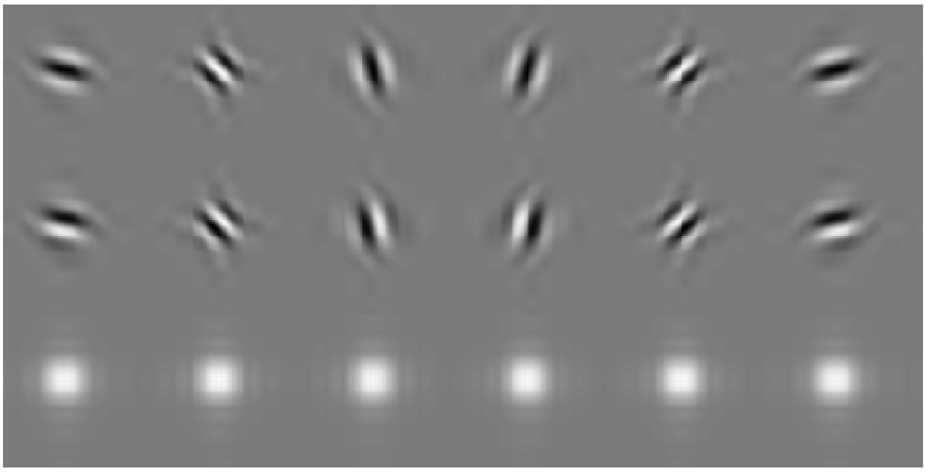

Fig. 5: Directional complex wavelets for 2D DWT

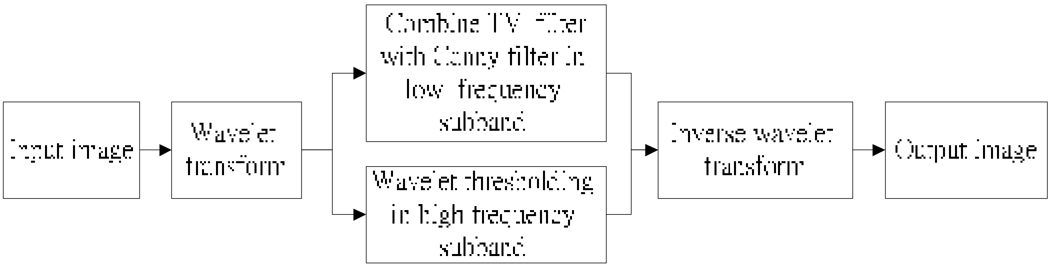

Fig. 6: Flowchart of proposed method

Fig. 7(a): Original image

Fig. 7(b): Noise image

Fig. 8(a): Reconstructive image by separable DWT

Fig. 8(b): Reconstructive image by complex dual-tree DWT

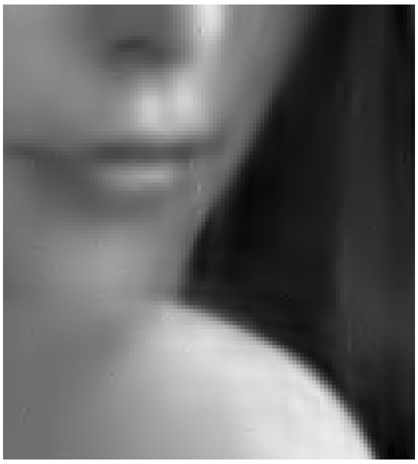

Fig. 9(a): Reconstructive image by complex dual-tree DWT

Fig. 9(b): Reconstructive image by real dual-tree DWT