以 T-S 模糊模型建構之模糊控制器應用於非線性系統

Fuzzy Controllers for Nonlinear Systems via T-S Fuzzy Models

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摘要

本文討論在外力干擾下被動調質阻尼與主動模糊控制減振的效用。一般而言調質阻尼在線性系統效果很好。在此，我們提出模糊控制的方法應用於非線性的情況。利用李雅普諾夫直接法(Lyapunov's direct method)推導一穩定準則以確保非線性系統達到穩定。文中提出平行分散補償(Parallel Distributed Compensation, PDC)的控制技巧，藉此架構吾人將設計一模糊控制器以穩定非線性之非線性調質阻尼結構。文末，舉一例藉由數值模擬證實理論可行。

關鍵字：調質阻尼，T-S 模糊模型，模糊控制。

Abstract

This paper investigates the effectiveness of a passive Tuned Mass Damper (TMD) and active fuzzy controllers in reducing the structural responses under the external force. In general, TMD is good for linear system. We proposed here a fuzzy controller to deal with the nonlinear system. For the fuzzy controller, a stability criterion in terms of Lyapunov's direct method is derived to guarantee the stability of TMD systems. Based on the decentralized control scheme and this criterion, a set of model-based fuzzy controllers is then synthesized via the technique of parallel distributed compensation (PDC) to stabilize nonlinear TMD systems. Finally, an example is given to illustrate the concepts discussed throughout this paper.

Key words: TMD, T-S fuzzy models, fuzzy control.

I. Introduction

Traditional structural design depends on structural strength and capability to dissipate energy due to dynamic forces such as machine loading, wind forces and earthquakes. The use of passive tuned mass dampers (TMDs) as a means to control and reduce the vibration of dynamic systems was first proposed by Frahm in 1909 [1]. Since then, much research has been done to investigate the control effectiveness of passive TMDs [2-4]. These articles show that the TMDs are suitable for a linear resonant system and it will be useful only for the frequency of TMD close to the primary structure [5]. Nevertheless, for a relatively small displacement, the restoring force of the spring can be modeled linearly. Nonlinear stiffness is considered for a large displacement so that the TMD is not appropriate [6]. The objective of this paper is to derive a stability criterion for model-based fuzzy controller to guarantee the uniformly ultimately bounded (UUB) stable of nonlinear systems.

In the past few years, fuzzy-rule-based modeling has become an active research field because of its unique merits in solving complex nonlinear system identification and control problems. In attempt to attain more flexibility and more effective capability of handling and processing uncertainties in complicated and ill-defined systems, Zadeh [7] proposed a linguistic approach as the model of human thinking, which introduced the fuzziness into systems theory [8]. Unlike traditional modeling, fuzzy rule-based modeling is essentially a multimodel approach in which individual rules are combined to describe the global behavior of the system [9].

There have been many successful applications in fuzzy control in recent years. In spite of the success, there are still many basic issues that remain to be further addressed. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems. Recently, there have been significant research efforts on these issues [10-15]. However, as far as we know, the stabilization problem of nonlinear TMD systems remains unresolved.

Hence, a stability criterion in terms of Lyapunov's direct method is derived in this study to guarantee the stability of TMD systems. According to this criterion and the control scheme, a model-based fuzzy controller is then synthesized to stabilize the nonlinear TMD system. Moreover, the system is represented by a Takagi-Sugeno (T-S) type fuzzy model. In this type of fuzzy model, each fuzzy implication is expressed by a linear system model, which allows us to use linear feedback control as in the case of feedback stabilization. The control design is carried out based on the fuzzy model via the parallel distributed compensation (PDC) scheme. The idea is that a linear feedback control is designed for each local linear model. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller [10, 13]. In other words, a stability criterion in terms of Lyapunov's direct method is derived to guarantee the stability of systems. Based on this criterion and the control scheme, a model-based fuzzy controller is to stabilize the nonlinear system.

This paper is organized as follows. First, the T-S fuzzy model is briefly reviewed and the system description is presented. Then, a stability criterion is derived to guarantee the stability of systems. Next, a TMD is used to reduce the vibration of dynamic linear system but it fails for the nonlinear system. So, a set of model-based fuzzy controllers via the technique of PDC is proposed to stabilize the nonlinear TMD system. Finally, a numerical example of nonlinear TMD system with simulations is given to illustrate the results, and the conclusions are drawn.

II. System Description

Consider a nonlinear system ***N*** composed of J subsystems N_j, $j = 1, 2, \cdots, J$. The jth subsystem N_j is described as follows:

$$\dot{x}_j(t) = f_j(x_j(t), u_j(t)) + \sum_{\substack{n=1 \\ n \neq j}}^{J} b_{nj}(x_n(t)) + \phi_j(t) \qquad (2.1)$$

where f_j is the nonlinear vector-valued function, $x_j(t)$ is the state vector, $u_j(t)$ is the input vector, $\phi_j(t)$ denotes the external force and b_{nj} is the nonlinear interconnection between the nth and jth subsystems.

Definition 2.1 [6]**:** The solution of a dynamic system are said to be uniformly ultimately bounded (UUB) if there exist positive constants ς and κ, and for every $\delta \in (0, \kappa)$ there is a positive constant $T = T(\delta)$, such that

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| \le \varsigma, \ \forall t \ge t_0 + T$$

In a little more than a decade ago, a fuzzy dynamical model had been developed primarily from the pioneering work of Takagi and Sugeno [16] to represent local linear input/output relations of nonlinear systems. This dynamical model is described by fuzzy IF-THEN rules and it will be employed here to handle the control design problem of the nonlinear interconnected system ***N***. The ith rule of this fuzzy model for the nonlinear interconnected subsystem N_j is proposed as the following form:

Rule i: IF $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj} THEN

$$\dot{x}_j(t) = A_{ij}x_j(t) + \sum_{\substack{n=1 \\ n \neq j}}^{J} \hat{A}_{inj}x_n(t) + B_{ij}u_j(t) + \phi_j(t) \quad (2.2)$$

where $x_j^T(t) = [x_{1j}(t), x_{2j}(t), \cdots, x_{gj}(t)] \in R^{1\times g}$ denotes the state vector,

$u_j^T(t) = [u_{1j}(t), u_{2j}(t), \cdots, u_{mj}(t)] \in R^{1\times m}$ denotes the control input,

$\phi_j^T(t) = [\phi_{1j}(t), \phi_{2j}(t), \cdots, \phi_{zj}(t)] \in R^{1\times z}$ denotes the unknown disturbances with a known upper bound $\phi_{upj}(t) \ge \|\phi_j(t)\|$. $i = 1, 2, \cdots, r_j$ and r_j is the number of IF-THEN rules; A_{ij}, $\hat{A}_{inj}$ and B_{ij} are constant matrices with appropriate dimensions; M_{ipj} ($p = 1, 2, \cdots, g$) are the fuzzy sets, and $x_{1j}(t) \sim x_{gj}(t)$ are the premise variables. The final state of this fuzzy dynamic model is inferred as follows:

$$\dot{x}_j(t) = \frac{\sum_{i=1}^{r_j} w_{ij}(t)[A_{ij}x_j(t) + \sum_{\substack{n=1 \\ n \neq j}}^{J} \hat{A}_{inj}x_n(t) + B_{ij}u_j(t) + \phi_j(t)]}{\sum_{i=1}^{r_j} w_{ij}(t)}$$

$$= \sum_{i=1}^{r_j} h_{ij}(t)(A_{ij}x_j(t) + \sum_{\substack{n=1 \\ n \neq j}}^{J} \hat{A}_{inj}x_n(t) + B_{ij}u_j(t)) + \phi_j(t) \quad (2.3)$$

with

$$w_{ij}(t) \equiv \prod_{p=1}^{g} M_{ipj}(x_{pj}(t)), \quad h_{ij}(t) \equiv \frac{w_{ij}(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} \qquad (2.4)$$

in which $M_{ipj}(x_{pj}(t))$ is the grade of membership of $x_{pj}(t)$ in M_{ipj}. In this paper, it is assumed that $w_{ij}(t) \ge 0$, $i = 1, 2, \cdots, r_j$; $j = 1, 2, \cdots, J$ and $\sum_{i=1}^{r_j} w_{ij}(t) > 0$ for all t. Therefore, $h_{ij}(t) \ge 0$ and $\sum_{i=1}^{r_j} h_{ij}(t) = 1$ for all t.

In the next section, the concept of PDC scheme is utilized to design fuzzy controllers.

III. Parallel Distributed Compensation

According to the decentralized control scheme, a set of model-based fuzzy controllers is synthesized via the technique of parallel distributed compensation (PDC) to stabilize the nonlinear system ***N***. The concept of PDC scheme is that each control rule is distributively designed for the corresponding rule of a T-S fuzzy model. The fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts [11]. Since each rule of the fuzzy model is described by a linear state equation, a linear control theory can be used to design the consequent parts of a fuzzy controller. The resulting overall fuzzy controller, nonlinear in general, is achieved by fuzzy blending of each individual linear controller.

Hence, the *j*th model-based fuzzy controller can be described as follows:

Rule *i*: IF $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj}

THEN $u_j(t) = -K_{ij} x_j(t)$, (3.1)

where $i = 1, 2, \cdots, r_j$. The final output of this fuzzy controller is

$$u_j(t) = -\frac{\sum_{i=1}^{r_j} w_{ij}(t) K_{ij} x_j(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} = -\sum_{i=1}^{r_j} h_{ij}(t) K_{ij} x_j(t). \quad (3.2)$$

Substituting Eq. (3.2) into Eq. (2.1) we have the *j*th ($j = 1, 2, \cdots, J$) closed-loop subsystem $\overline{F}_j$:

$$\dot{x}_j(t) = \sum_{i=1}^{r_j}\sum_{l=1}^{r_j} h_{ij}(t) h_{lj}(t)[(A_{ij} - B_{ij}K_{lj}) x_j(t)] + \sum_{\substack{n=1 \\ n\neq j}}^{J} \hat{A}_{inj} x_n(t) + \phi_j(t)$$

. (3.3)

In the following, a stability criterion is proposed to guarantee the stability of the closed-loop fuzzy system $\overline{\boldsymbol{F}}$ which consists of *J* closed-loop subsystems described in Eq. (3.3). Prior to examination of stability of $\overline{\boldsymbol{F}}$, an useful concept is given below.

Lemma 3.1 [17]**:** For real matrices *A* and *B* with appropriate dimensions, we have

$$A^T B + B^T A \le \sigma A^T A + \sigma^{-1} B^T B$$

where σ is a positive constant.

Theorem 3.1: The closed-loop fuzzy system $\overline{\boldsymbol{F}}$ is stable, if there exist symmetric positive definite matrices P_j and positive constants α, γ and the feedback gains K_{ij}'s shown in Eq. (3.2) are chosen to satisfy

(I) $\hat{\lambda}_{inj} = \lambda_M(Q_{inj}) < 0$

for $i = 1, 2, \cdots, r_j$; $n, j = 1, 2, \cdots, J$ (3.4a)

$\tilde{\lambda}_{ilnj} = \lambda_M(Q_{ilnj}) < 0$

for $i < l \le r_j$; $n, j = 1, 2, \cdots, J$ (3.4b)

or

(II)

$$\Lambda_j = \sum_{n=1}^{J} \begin{bmatrix} \hat{\lambda}_{1nj} & \tilde{\lambda}_{12nj} & \cdots & \tilde{\lambda}_{1r_jnj} \\ \tilde{\lambda}_{12nj} & \hat{\lambda}_{2nj} & \cdots & \tilde{\lambda}_{2r_jnj} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\lambda}_{1r_jnj} & \tilde{\lambda}_{2r_jnj} & \cdots & \hat{\lambda}_{r_jnj} \end{bmatrix} < 0 \quad \text{for}$$

$j = 1, 2, \cdots, J$ (3.5)

where

$$Q_{inj} = \{\frac{1}{J}[(A_{ij} - B_{ij}K_{ij})^T P_j + P_j(A_{ij} - B_{ij}K_{ij})]$$
$$+\alpha^{-1} P_j \hat{A}_{inj} \hat{A}_{inj}^T P_j + \alpha(\frac{J-1}{J}) I + \frac{1}{J}(\gamma^{-1} P_j^2)\}, \quad (3.6)$$

$$Q_{ilnj} = \{\frac{1}{J}[(H_{ilj}^T P_j + P_j H_{ilj})]$$
$$+\alpha^{-1} P_j \hat{A}_{inj} \hat{A}_{inj}^T P_j + \alpha(\frac{J-1}{J}) I + \frac{1}{J}(\gamma^{-1} P_j^2)\}, \quad (3.7)$$

with $H_{ilj} = \dfrac{(A_{ij} - B_{ij}K_{lj}) + (A_{lj} - B_{lj}K_{ij})}{2}$.

(3.8)

Moreover, $\lambda_M(Q_{inj})$ and $\lambda_M(Q_{ilnj})$ denote the maximum eigenvalues of Q_{inj} and Q_{ilnj}, respectively.

Remark 3.1: In principle, both the condition **(I)** and condition **(II)** can be used to test the stability of the closed-loop fuzzy system $\overline{\boldsymbol{F}}$. It is therefore reasonable to check the stability with either one of the conditions and, if it fails, then resort to the other.

IV. Examples

4.1 TMD system: A passive TMD mounted on a shear structure is modeled as a two-degree-of freedom structure-TMD system as shown in Fig. 4.1. The parameters m_1, c_1 and k_1 represent mass, damping and stiffness in the subsystem 1; m_2, c_2 and k_2 represent mass, damping and stiffness in the subsystem 2; *f* and *u* present external force and control input. The equation of motion with no control input can be written as [5]

$$\begin{cases} \ddot{s}_1 + 2\xi_1\omega_1\dot{s}_1 - 2\mu\xi_2\omega_2(\dot{s}_2 - \dot{s}_1) + \omega_1^2 s_1 - \mu\omega_2^2(s_2 - s_1) = f \\ \ddot{s}_2 + 2\xi_2\omega_2(\dot{s}_2 - \dot{s}_1) + \omega_2^2(s_2 - s_1) = 0 \end{cases}$$

(4.1)

where

$\omega_1 (= \sqrt{k_1/m_1})$ is natural frequency of primary structure ;

$\omega_2 (= \sqrt{k_2/m_2})$ is natural frequency of TMD

$\xi_1 (= c_1/2m_1\omega_1)$ is damping ratio of primary structure ;

$\xi_2 (= c_2/2m_2\omega_2)$ is damping ratio of TMD

$\mu (= m_2/m_1)$ denotes mass ratio of TMD to primary structure ; $\beta (= \overline{\omega}/\omega_1)$ denotes frequency ratio

$\overline{\omega}$ denotes frequency of external force

Fig. 4.2 shows the effectiveness of a TMD system in reducing the response due to an external force with $m_1 = 1,\ \omega_1 = \omega_2 = 1.29,\ c_1 = 2.506\times10^{-3},\ \xi_2 = 2.506 \times10^{-5},\ f = \cos(\varpi\ \mathrm{t}),\ \mu = 0.01,\ \varpi = 1.29$ and initial conditions $s_1(0) = \dot{s}_1(0) = s_2(0) = \dot{s}_2(0) = 0$. Fig. 4.3 shows the dynamic magnification factor where restoring force is a linear function. So, the passive TMD is appropriate when the frequency of external excitation is close to the structure. But, the restore force of spring stiffness is nonlinear in actual systems. It is no use for the TMD system shown in Figs. 4.4-4.6 with $k_1 = 1.664(1 - a^2 s_1^2)$, $k_2 = 0.01664(1 - a^2 s_2^2)$ and initial conditions $s_1(0) = \dot{s}_1(0) = s_2(0) = \dot{s}_2(0) = 0$ [6, 18]. A method for fuzzy controller is proposed to guarantee the stability of nonlinear system in next section.

4.2 PDC fuzzy controllers: The objective of this section is to synthesize a set of T-S fuzzy controllers such that the nonlinear interconnected system $\boldsymbol{N}$ which is composed of two subsystems described in Eq. (4.1) with nonlinear $k(x)$ of a = 0.01 can be stabilized.

Subsystem 1:

$$\begin{cases} \dot{x}_{11}(t) = 10x_{21}(t) \\ \dot{x}_{21}(t) = -0.1681\,x_{11}(t) + 1.6641\times10^{-7}\ x_{11}^3(t) \\ \qquad -2.531\times10^{-3}\ x_{21}(t) + 1.6641\times10^{-3}\ x_{12}(t) \\ \qquad -1.6641\times10^{-9}\ x_{12}^3(t) + 1.6641\times10^{-9} x_{11}(t)x_{12}^2(t) \\ \qquad +2.506\times10^{-5}\ x_{22}(t) + \cos(1.29\,t) + 5\ u_1(t) \end{cases} \tag{4.2}$$

Subsystem 2:

$$\begin{cases} \dot{x}_{12}(t) = 10\,x_{22}(t) \\ \dot{x}_{22}(t) = -0.16641\,x_{12}(t) + 1.6641\times10^{-7}\ x_{12}^3(t) \\ \qquad -2.506\times10^{-3}\ x_{22}(t) + 0.16641\,x_{11}(t) \\ \qquad -1.664\times10^{-7}\ x_{11}(t)x_{12}^2(t) \\ \qquad +2.506\times10^{-3}\ x_{21}(t) + 4.5\,u_2(t) \end{cases} \tag{4.3}$$

Where $x_{11} = 10\,s_1$, $x_{21} = \dot{s}_1$, $x_{12} = 10\,s_2$ and $x_{22} = \dot{s}_2$. How do we synthesize three T-S fuzzy controllers to stabilize the nonlinear interconnected system $\boldsymbol{N}$?

Solution: We can solve this problem according to the following steps.

Step 1: Establish a T-S fuzzy model for each nonlinear interconnected subsystem. To minimize the design effort and complexity, we try to use as few rules as possible. Hence, the subsystems $(4.2-4.3)$ are approximated with the following fuzzy models:

T-S fuzzy model of subsystem 1:

Rule 1: IF $x_{11}(t)$ is M_{111}

$$\text{THEN}\quad \dot{x}_1(t) = A_{11}x_1(t) + \sum_{\substack{n=1\\ n\neq j}}^{2} \hat{A}_{1n1}x_n(t) + B_{11}u_1(t),$$

Rule 2: IF $x_{11}(t)$ is M_{211}

$$\text{THEN}\quad \dot{x}_1(t) = A_{21}x_1(t) + \sum_{\substack{n=1\\ n\neq j}}^{2} \hat{A}_{2n1}x_n(t) + B_{21}u_1(t)$$

where

$$x_1^T(t) = [x_{11}(t)\ \ x_{21}(t)],\quad A_{11} = \begin{bmatrix} 0 & 10 \\ -0.1681 & -0.0025 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0 & 10 \\ -0.1680 & -0.0025 \end{bmatrix},\ \hat{A}_{121} = \begin{bmatrix} 0 & 0 \\ 0.0017 & 0.00003 \end{bmatrix},$$
$$\hat{A}_{221} = \begin{bmatrix} 0 & 0 \\ 0.0016 & 0.00003 \end{bmatrix},\ B_{11} = \begin{bmatrix} 0 \\ 5 \end{bmatrix},\ B_{21} = \begin{bmatrix} 0 \\ 5 \end{bmatrix} \tag{4.4}$$

and the membership functions for Rule 1 and Rule 2 are

$$M_{111}(x_{11}(t)) = \frac{1}{\left[1 + \left|\frac{1 - x_{11}(t)}{2}\right|\right]^2},$$

$$M_{211}(x_{11}(t)) = 1 - M_{111}(x_{11}(t)).$$

T-S fuzzy model of subsystem 2:

Rule 1: IF $x_{12}(t)$ is M_{112}

$$\text{THEN}\quad \dot{x}_2(t) = A_{12}x_2(t) + \sum_{\substack{n=1\\ n\neq j}}^{2} \hat{A}_{1n2}x_n(t) + B_{12}u_2(t)$$

Rule 2: IF $x_{12}(t)$ is M_{212}

$$\text{THEN}\quad \dot{x}_2(t) = A_{22}x_2(t) + \sum_{\substack{n=1\\ n\neq j}}^{2} \hat{A}_{2n2}x_n(t) + B_{22}u_2(t)$$

where

$$x_2^T(t) = [x_{12}(t)\ \ x_{22}(t)],\quad A_{12} = \begin{bmatrix} 0 & 10 \\ -0.1664 & -0.0025 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} 0 & 10 \\ -0.1663 & -0.0025 \end{bmatrix},\ \hat{A}_{112} = \begin{bmatrix} 0 & 0 \\ 0.1664 & 0.0025 \end{bmatrix},$$
$$\hat{A}_{212} = \begin{bmatrix} 0 & 0 \\ 0.1663 & 0.0025 \end{bmatrix},\ B_{12} = \begin{bmatrix} 0 \\ 4.5 \end{bmatrix},\ B_{22} = \begin{bmatrix} 0 \\ 4.5 \end{bmatrix} \tag{4.5}$$

and membership functions for Rule 1 and Rule 2 are

$$\begin{cases} M_{112}(x_{12}(t)) = \dfrac{2}{3\pi}x_{12}(t) + 1 & \text{when } -\dfrac{3\pi}{2} \le x_{12}(t) \le 0 \\ M_{112}(x_{12}(t)) = -\dfrac{2}{3\pi}x_{12}(t) + 1 & \text{when } 0 < x_{12}(t) \le \dfrac{3\pi}{2} \\ M_{112}(x_{12}(t)) = 0 & \text{otherwise,} \end{cases}$$

$$M_{212}(x_{12}(t)) = 1 - M_{112}(x_{12}(t)).$$

Step 2: In order to stabilize the fuzzy interconnected system $\boldsymbol{F}$, two model-based fuzzy controllers designed via the concept of PDC scheme are synthesized as follows.

Fuzzy controller of subsystem 1:

Rule 1: IF $x_{11}(t)$ is M_{111}

THEN $u_1(t) = -K_{11}x_1(t)$

Rule 2: IF $x_{11}(t)$ is M_{211}

THEN $u_1(t) = -K_{21}x_1(t)$. (4.6)

Fuzzy controller of subsystem 2:

Rule 1: IF $x_{12}(t)$ is M_{112}

THEN $u_2(t) = -K_{12}x_2(t)$,

Rule 2: IF $x_{12}(t)$ is M_{212}

THEN $u_2(t) = -K_{22}x_2(t)$. (4.7)

Step 3: To meet the stability condition **(I)** or condition **(II)** of Theorem 3.1, the matrices Q_{inj}'s in Eq. (3.6) are chosen to be negative definite. Hence, based on Eqs. (4.4 –4.7), we can obtain the following positive definite matrices P_j (j =1, 2) and K_{ij}'s via LMI optimization algorithms such that the matrices Q_{inj}'s are negative definite with $\alpha = 0.1$ and $\gamma = 0.1$:

$$P_1 = \begin{bmatrix} 0.1233 & 0.0461 \\ 0.0461 & 0.0427 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.1063 & 0.0814 \\ 0.0814 & 0.1007 \end{bmatrix}, \tag{4.8}$$

$$K_{11} = [11.9664 \quad 9.9995], \quad K_{21} = [7.4664 \quad 7.9995],$$
$$K_{12} = [6.6297 \quad 7.7772], \quad K_{22} = [3.2964 \quad 5.5550]. \tag{4.9}$$

Substituting Eqs. (5.4 – 5.9) into Eqs. (3.6 – 3.7) yields

Q_{inj}'s <0 and $Q_{i\ln j}$'s <0 (4.10)

and the eigenvalues of them are given below:

$$\lambda(\Lambda_1) = 0.0036, \ -1.1287$$
$$\lambda(\Lambda_2) = 0.0100, \ -0.5285. \tag{4.11}$$

Although the matrices Λ_j ($j = 1, 2$) are not all negative definite, the inequality (3.4) is satisfied. Therefore, based on condition (I) of Theorem 3.1, the T-S fuzzy controllers described in Eqs. (4.6 – 4.7) can stabilize the fuzzy interconnected system $\boldsymbol{F}$. To assess the effectiveness of the T-S fuzzy controllers, we apply the same T-S fuzzy controllers to the nonlinear interconnected TMD system $\boldsymbol{N}$ which consists of two subsystems described in Eqs. (4.2 – 4.3). Simulation results of each closed-loop subsystem $\overline{N}_j$ ($j = 1, 2$) are illustrated in Figs. 4.7 – 4.8 with initial conditions, $x_{11}(0) = 1$, $x_{21}(0) = -1$, $x_{12}(0) = 0.1$ and $x_{22}(0) = -0.1$.

VI. Conclusions

In order to ensure the stability of interconnected systems, a stability criterion is derived from Lyapunov's direct method. According to this criterion and the decentralized control scheme, a set of model-based fuzzy controllers is synthesized to stabilize the nonlinear interconnected TMD system. Hence, the proposed fuzzy control can be applied to any robust control design of nonlinear interconnected systems. Finally, a numerical example with simulations is provided to demonstrate the results.

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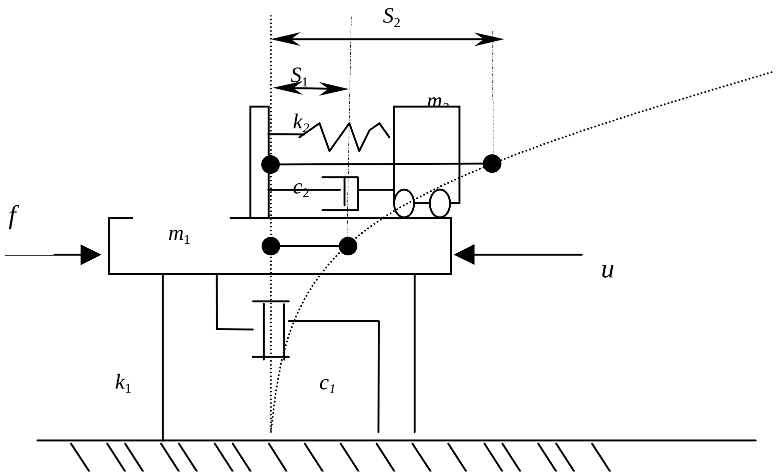

Fig. 4.1. Two-DOF structure-TMD system.

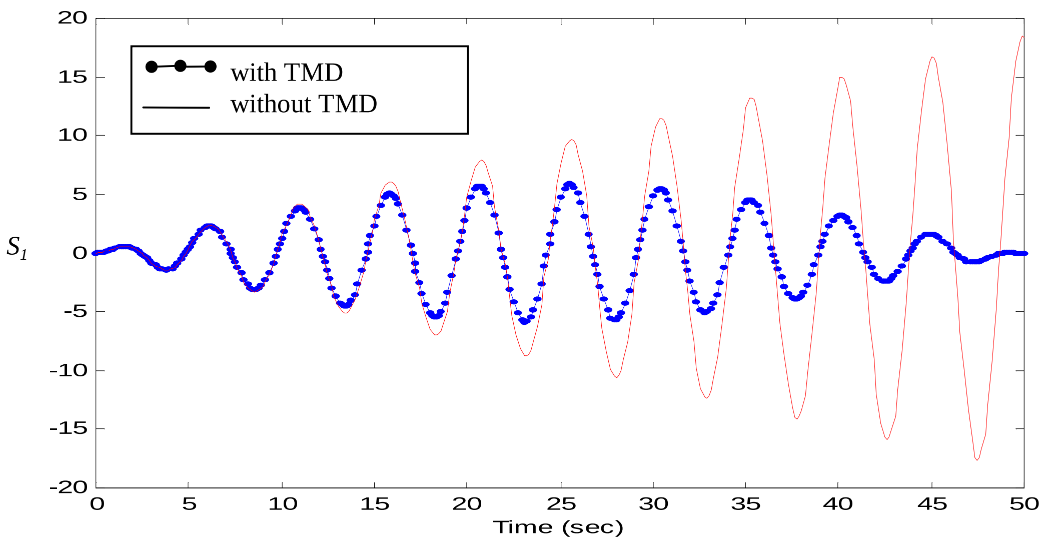

Fig. 4.2. The effectiveness of a TMD system.

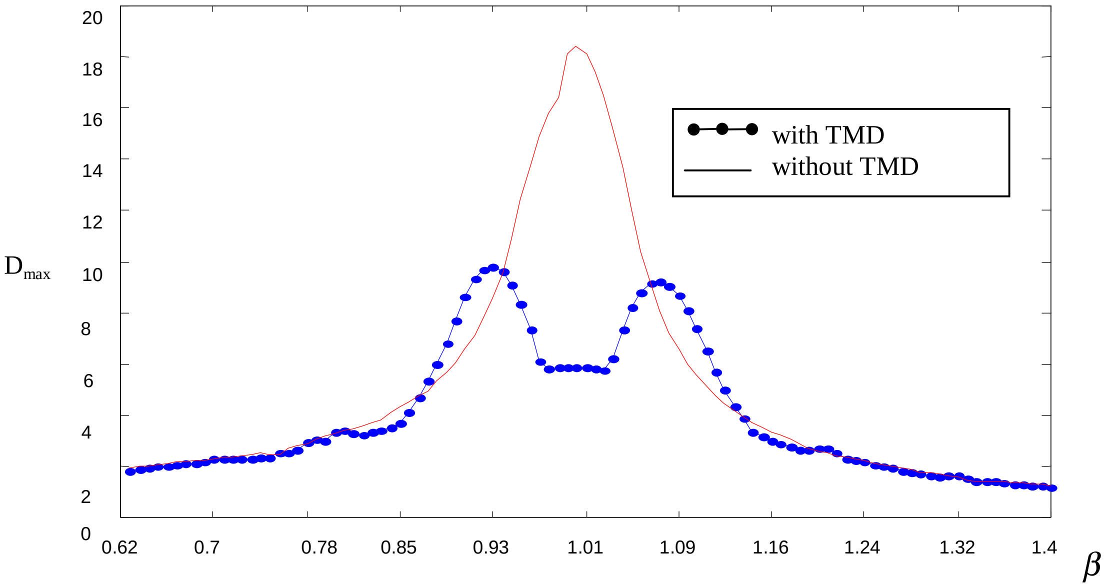

Fig. 4.3. The effectiveness of a TMD system with linear stiffness *k*(x).

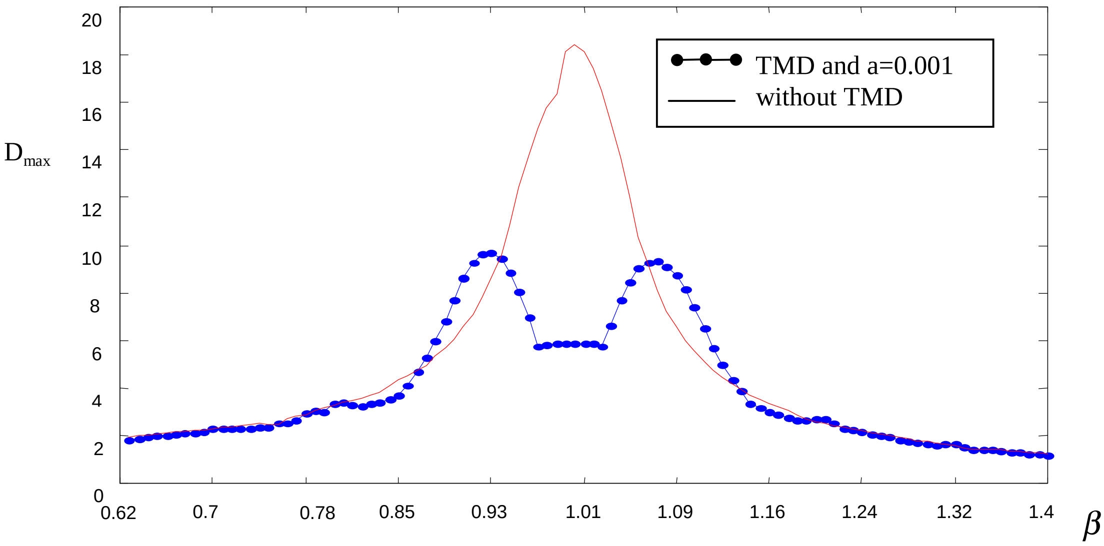

Fig. 4.4. Dynamic magnification factor of a TMD system with nonlinear stiffness *k*(x).

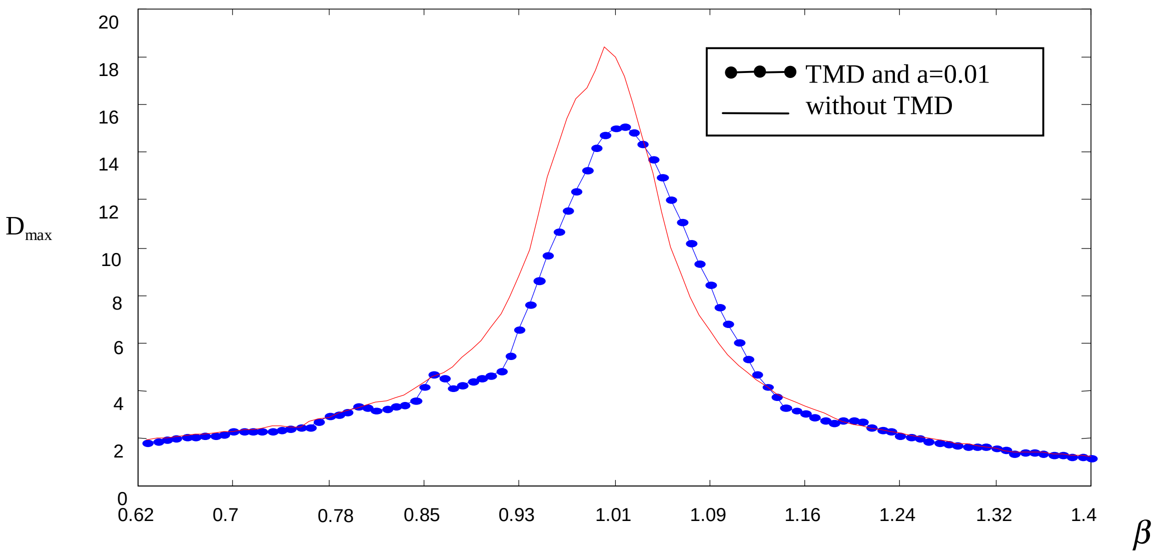

Fig. 4.5. Dynamic magnification factor of a TMD system with nonlinear stiffness *k*(x).

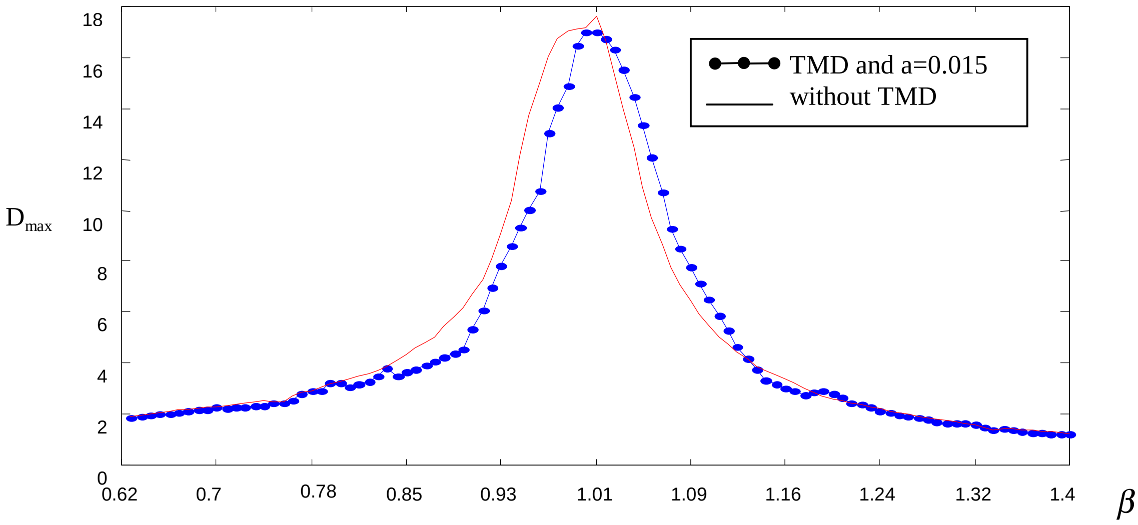

Fig. 4.6. Dynamic magnification factor of a TMD system with nonlinear stiffness *k*(x).

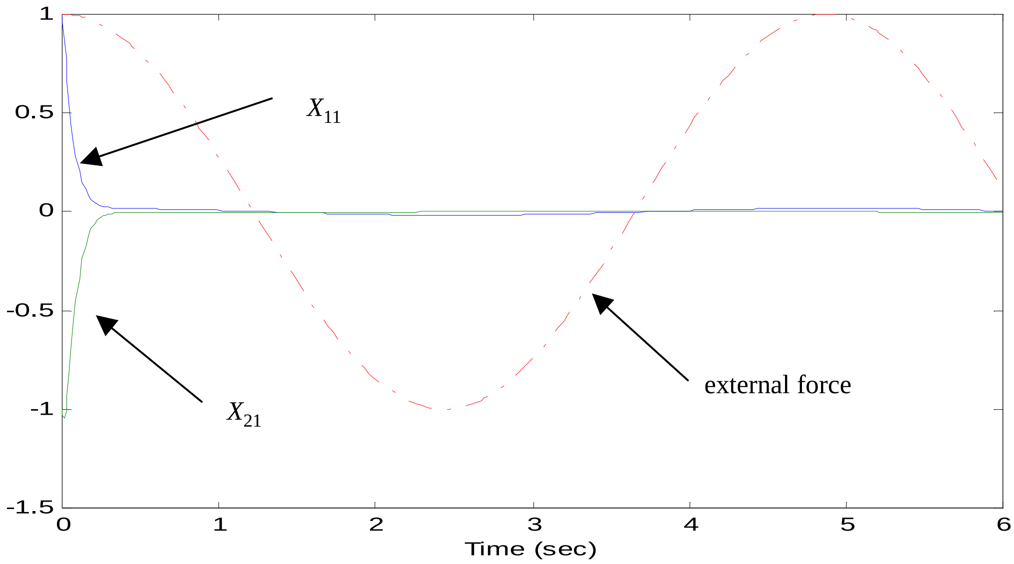

Fig. 4.7. The state response of subsystem 1.

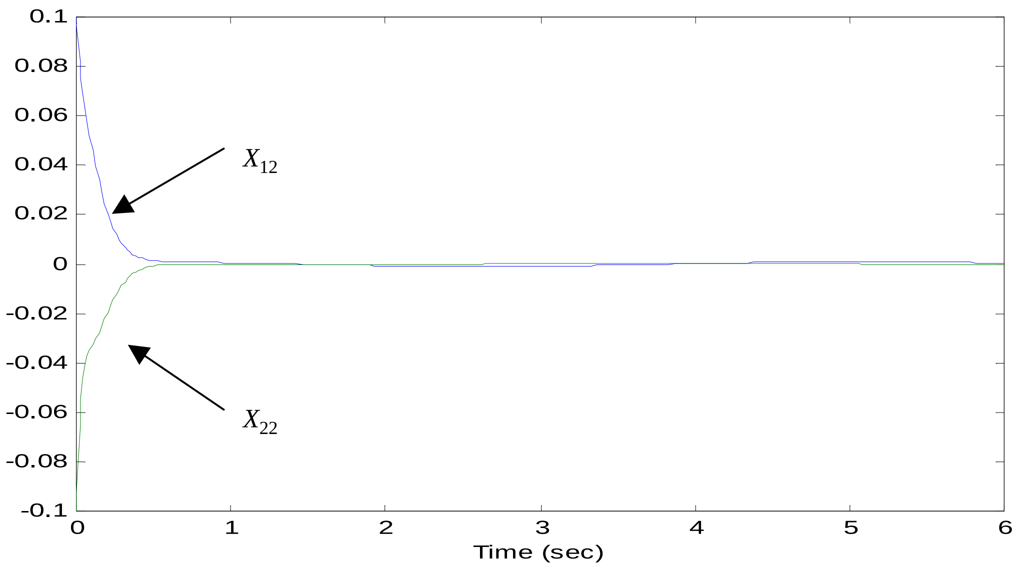

Fig. 4.8. The state response of subsystem 2.