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Liquidity-Adjusted Value-at-Risk for TWSE Leverage/ Inverse ETFs: A Hellinger Distance Measure Research

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This paper empirically investigates the liquidity-adjusted Value-at-Risk (LaVaR) of TWSE Leverage/Inverse ETFs using the Hellinger distance measure by sensitizing endogenous liquidity risk with trade sizes at 1%, 3%, and 6%. By incorporating adjusted exogenous and endogenous liquidity risk, we find that LaVaR produces more accurate risk estimates and increases with trade size. The practical failure rates of all ETFs are largely consistent with their theoretical failure rates. Despite the use of different empirical models, China ETFs have a higher risk level than Taiwan ETFs in both bullish and bearish markets.

Keywords: LaVaR, TWSE leverage/inverse ETFs, hellinger distance measure, exogenous liquidity risk, endogenous liquidity risk

JEL classification: G32, D81, C58

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1 Introduction

Thirty different domestic exchange traded funds (ETFs) have been listed on the Taiwan Securities Exchange (TWSE) in the past 10 years. On October 24, 2014, TWSE introduced Leverage/Inverse ETFs, which are umbrella mutual funds, specifically aimed at the China market in response to incremental expansion in global market investment.[1] These innovative instruments include Taiwan's domestic Daily 50 Bull/Bear ETFs (Taiwan 50 Leverage/Inverse ETFs hereafter) and Greater China Region Shanghai Stock Exchange Constituent 180 Leverage/Inverse ETFs (China SSE 180 Leverage/Inverse ETFs hereafter).[2,3] This research investigates Taiwan's ETF market through two TWSE Leverage/Inverse ETFs, which respectively and passively track the Taiwan 50 index and the China SSE 180 index. Because the issued and trading value analysis statistical data about Leverage/Inverse ETFs are from the Bulletin Board Download Page of TWSE and FSC Annual Reports and the market transaction analysis is based on annual data of Taiwan Money DJ website, we thus choose the empirical research period of October 24, 2014 to the end of 2015, i.e., the 3Q of 2014 and whole 2015.

Leveraged and inverse ETFs enable investors to invest more aggressively in bullish and bearish markets, respectively. They offer potential multiple (e.g., double or triple) returns over the index they track, while their calculation and performance vary over longer time periods as measured by continuous compounding following each daily return. Leverage/Inverse ETFs can magnify the index returns and operate following different price movements, making them increasingly popular among investors. To observe the market transaction information summarized through the end of 2015, the total accumulated trading value of Taiwan Leverage/Inverse ETFs for the 14-month period (i.e., October 24 2014 to December 31, 2015) is NT$663 billion (see Table 1). On the whole, the China SSE 180 Leverage/Inverse ETFs have

[1]TWSE Leverage/Inverse ETFs are umbrella mutual funds designed to contain two alternative funds of a similar nature for investors to choose, including Leverage and Inverse ETFs. The Leverage ETF can operate in bullish markets with a multiple of 2x the index return, while the Inverse ETF can operate in bearish markets with a multiple of -1x the index return.

[2]Taiwan 50 Leverage/Inverse ETFs are issued by Yuanta Financial Holding Company (Yuanta FHC) and listed on the TWSE on October 24, 2014.

[3]China SSE 180 Leverage/Inverse ETFs are issued by Fubon Financial Holding Company (Fubon FHC) and listed on the TWSE on November 11, 2014.

a higher risk level than the Taiwan 50 Leverage/Inverse ETFs. Compared with two Leverage/Inverse ETFs in different price trends, the China SSE 180 Leverage and Inverse ETFs have a higher risk level in bullish and bearish markets than the risk of Taiwan 50 Leverage and Inverse ETFs. In 2014, Leverage/Inverse ETFs accounted for 8.78% issued, and rising incrementally to 61.02% by 2015, while nearly 51% of all ETFs' funds were invested in overseas markets in 2015 (see Table 2). The TWSE aims to attract investors to keep their funds in Taiwan rather than directly investing abroad (e.g., particularly in China's stock market). The instrument design focuses mainly on overseas markets and differs from initial issues on the Korea Exchange (KRX) and Tokyo Stock Exchange (TSE), which both emphasize domestic stock indices and markets (e.g., the first ETF issued on KRX in 2009 was the Samsung KODEX Inverse ETF, while the first issued ETF on TSE in 2012 was the TOPIX Leverage/Inverse ETF).

Table 1. Leverage/Inverse ETF's Market Transaction Analysis

ETFs / Item	Taiwan 50 ETF		China SSE 180 ETF	
	Leverage	Inverse	Leverage	Inverse
Initial listing	TWSE	TWSE	SSE	SSE
Fund size	NT$1.485 billion	NT$16.581 billion	NT$21.658 billion	NT$1.766 billion
Trading value	NT$24.99 billion	NT$73.01 billion	NT$505.25 billion	NT$59.38 billion
Issue price	20	20	20	20
High price	24.91	22.88	69.50	14.67
Low price	14.57	17.64	22.35	9.04
Annual average return	-15.39%	3.14%	8.10%	-5.59%
Annual Std. Dev.	25.50%	13.62%	69.58%	59.06%
Beta	1.76	-0.92	1.83	-0.70
Sharpe ratio	-0.08%	0%	0.06%	-0.27%

Source: http://www.moneydj.com. (April 1, 2016)

The percentage trading size (d_H hereafter) of ETFs in 2014 and 2015 respectively accounted for about 1% and 6% of total trading value in TWSE. With the gradual increase in ETF trading volume, risk-return controls and measures have become more important and indispensable for market participants. However, effective risk management is complex and requires sophisticated risk exposure analysis to formulate risk control plans. The Bank for International Settlements (BIS) classifies risks as belonging to one of five categories: market risk, credit risk,

Table 2. Leverage/Inverse ETFs' Market Issued and Trading Value

Year	No.	Percentage of all ETFs	Percentage of ETF investment markets		Percentage of Leverage/Inverse ETFs
			Domestic market	Overseas market	
2003	1	0.17%	100%	0%	-
2004	1	0.32%	100%	0%	-
2005	3	0.42%	100%	0%	-
2006	7	0.35%	100%	0%	-
2007	11	0.54%	100%	0%	-
2008	12	1.86%	90%	10%	-
2009	12	0.86%	85%	15%	-
2010	17	0.77%	79%	21%	-
2011	17	1.75%	74%	26%	-
2012	20	1.17%	74%	26%	-
2013	19	1.05%	71%	29%	-
2014	23	1.47%	56%	44%	8.78%
2015	30	9.12%	49%	51%	61.02%

Source: Bulletin Board Download Page on the Taiwan Financial Supervisory Commission (FSC). (April 1, 2016). Total trading value includes the transaction value of ETFs starting June 30, 2003 and includes the transaction value of Leverage/Inverse ETFs starting October 24, 2014.

operation risk, legal risk, and liquidity risk (see Table 3). In the 1988 Basel Accord (Basel I), to measure the amount of credit risk, the BASEL Committee on Banking Supervision (BSBS) provided a calculating method on the capital adequacy ratio. After the 1987 U.S. stock market crash, BSBC emphasized the importance of establishing a standard statistical model and quantitative analysis for market risk. Although the core measurements of market risk are from the mean-variance pricing model, banks and individual investors rely on more direct methods such as Value-at-Risk (VaR). In 1996, BCBS stressed the importance of market risk measures and required banks to declare their maximum threshold loss under fixed confidence levels and a given time horizon (see J. P. Morgan 1996 RiskMetrics measurement; U.S. Securities and Exchange Commission VaR information in 1997 financial statements exposure; VaR measure on market risk of 2004 Basel II Accord).[4,5] BCBS furthermore introduced three new directives in the 2004 Basel II Accords to cover exposure, operation risk, and legal risk. At the time, the relevant regulations were insufficient to adequately compare liquidity risk exposure and other types of risk, but a continuous series of crises (i.e., 1997 Asian and 2007-2008 global financial crises) resulted in ongoing low liquidity conditions that damaged equity markets worldwide. In the face of contagious liquidity risk, BCBS set rules to

[4]Data for the 2004 Basel II Accords are taken from http://www.bis.org.

[5]Data for the U.S. Securities and Exchange Commission VaR information in firms' 1997 financial statements are taken from https://www.sec.gov/divisions/corpfin/guidance/derivfaq.htm

regulate liquidity exposure in the Basel III Accords. These regulations require banks to report their liquidity coverage ratio (LCR) and net stable funding ratio (NSFR). However, these measures are still insufficient to adequately determine the liquidity risk effect on VaR (see the liquidity requirement of the 2010 Basel III Accords).[6]

Table 3. BCBS for Exposing and Assessing Different Investment Risks in Various Periods

Risk	Period
Liquidity risk	2010~2017 (Basel III)
Operation risk	2004 (Basel II)
Legal risk	2004 (Basel II)
Market risk	1996 (Basel I extended)
Credit risk	1988 (Basel I)

Source: Bulletin Board Download Page on the BCBS. (June 30, 2016)

Stoll (2000) points out that poor liquidity leads to friction costs in imperfect markets, thus creating a gap between theoretical and real market prices (Stoll, 2000; Berkowitz, 2000). However, traditional Value-at-Risk (VaR)[7] assumes the asset only has market risk, with no liquidity or credit risk. VaR could calculate the threshold loss value given a specific portfolio, time horizon, and one-tailed probability by mark-to-market pricing (Jorion, 2006; Chen *et al*., 2012; Chang *et al*., 2016). Bangia *et al*. (1999, 2001) introduce liquidity-adjusted Value-at-risk (LaVaR) and employ liquidity risk to adjust the VaR-only measure for the simple mean-variance at market risk, which is a basic traditional LaVaR model based on the imperfect market and frictional cost hypotheses. They note that disregarding the liquidity-adjusted VaR would underestimate risk by between 25% and 30%. Versus traditional VaR assuming good liquidity, LaVaR could measure the risk due to ill-liquidity effects. Bangia *et al*. (1999, 2001) also classify liquidity risk into exogenous liquidity risk and endogenous liquidity risk, where the bid-ask spread is a proxy variable to measure exogenous risk. The incrementally incurring spread gap directly correlates to the exogenous liquidity risk and increased cost of liquidity (COL) of a financial asset on traditional VaR (Aubier & Saout, 2002; Ourir & Snoussi, 2012).

Exogenous liquidity risk leads to mispricing between bid and ask prices and is seldom caused by individual investors, but always increases overall market price

[6]Data for the 2010 Basel III Accords are taken from http://www.bis.org.

[7] Value-at-risk (VaR) arose in 1993. By 1996, the amended Basel Accord required banks to comply with the contents to calculate VaR thresholds, by evaluating a 1% VaR model over a 12-month test period for 250 trading days (Chen *et al*., 2012). We take a 1% one-tailed probability and a 95% confidence levelto test consistency between practical violation rates and theoretical proportion of failures.

volatility and risk. Investors typically focus on uncontrollable risks and stress the exogenous liquidity risk effect, i.e. they look for spread volatility and appraise COL. Most investors exclusively target the exogenous liquidity risk effect. On the other hand, trade size is a proxy variable that measures endogenous risk (Bangia *et al*., 1999, 2001). The incrementally incurred trade size fluctuation directly correlates to the exogenous liquidity risk and increased transaction costs of a financial asset. In particular, higher exchange volumes sharply increase spread volatility and COL. Thus, increasing endogenous risk results in higher exogenous risk in advance and deepens market ill-liquidity in a vicious circle (Demsetz, 1968; Black, 1971a, 1971b; Kyle, 1985; Glosten & Harris, 1988; Stoll, 2000; Simonian, 2011).

Bangia *et al*. (1999, 2001) only define endogenous liquidity risk and explain possible effects on measuring VaR, but do not establish an empirical model and largely neglect its role in their empirical research. Lawrence & Robinson (1998), Häberle & Persson (2000), Aubier (2002), Zhan & Hun (2001), Shen *et al*. (2002), and Si & Fan (2012) use exogenous liquidity risk to refine the traditional VaR. Some early studies incorporate endogenous liquidity risk into LaVaR (e.g., Jarrow & Subramanian, 1997, 2012; Berkowitz, 2000; Subramanian & Jarrow, 2001; Cosandey, 2001; Le Saout, 2002), while Al Janabi (2011a, 2011b, 2013), Tsai & Li (2015), and Tsai & Wu (2016) employ trade size as an empirical variable to recalculate the liquidity horizon and measure endogenous liquidity risk effects. Simonian (2011) utilize trade size percentage (d_H) as a variable to adjust the endogenous liquidity risk effect on traditional LaVaR, assuming a numeric analysis at 1% market size and using the Hellinger distance measure concept in his research, which is a new method to adjust the endogenous liquidity risk on traditional LaVaR.

By combining the probability measure of the Hellinger distance characteristics in Bogachev's (2007) and Simonian's (2011) research,[8] our empirical study applies sensitivity analysis for measuring the endogenous liquidity risk effect. We assume there are two different probability distance measures between traditional LaVaR - one that considers only exogenous liquidity risk and the other involves endogenous

[8]The Hellinger distance $H(P, Q)$, also called the Bhattacharyya distance, can be used in metric space to measure the degree of disorder between two sets of probabilities in a d-metric state space Ω, e.g., probability P and Q. It monitors the probability measure space of P and Q. The Hellinger distance is defined in terms of the Hellinger integral, and we apply this concept to calculate the total variation distance. This distance measure provides a convenient expression of measures that fall in the range [0, 1] such as b and a. They are the same as the market position percentage of trade sizes between 0% and 100%.

liquidity risk-adjusted LaVaR. The two probability distribution functions f(x) and g(x) represent the two normal distributions in the state space Ω. In Lagarias *et al*. (1998), the probability distributions are normal distributions (N.D.) and are absolutely continuous with σ-finite dominating measure, which are $N(\mu_{s1}, \sigma_{s1})$ and $N(\mu_{s2}, \sigma_{s2})$ and the probability distributions of traditional LaVaR and endogenous-adjusted LaVaR, respectively. Here, $\mu_{s1,t}$ and $\sigma_{s1,t}$ are the parameters of probability distributions under exogenous liquidity risk (i.e., traditional LaVaR), $\mu_{s2,t}$ and $\sigma_{s2,t}$ are the parameters for probability distributions incorporating endogenous liquidity risk, and the probability density functions (PDF) of the normal distributions is $f(x|\mu_{si,t}, \sigma_{si,t}) = (2\pi)^{-0.5}\sigma_{si,t}^{-1} \exp\left\{-\frac{1}{2}\left[\left(x-\mu_{si,t}\right)/\sigma_{si,t}\right)\right]^2\right\}, i = 1,2$. We also regard the probability distance measure between the two probability distributions, where the Hellinger distance $H(P|Q)$ is equivalent to the percentage of market trade size (d_H), which is between 0% to 100%. By assuming the Hellinger distance's (d_H) different percentage of trade size at 0% as the traditional LaVaR case like in Bangis (1999, 2001), and at 1%, 3%, or 6%[9] when considering the endogenous-adjusted LaVaR, we revalue $\mu_{s2,t}$ and $\sigma_{s2,t}^2$ and then recalculate the new COL and the endogenous liquidity risk effect on traditional LaVaR.

The literature currently does not include relevant studies examining exogenous and endogenous liquidity risk together on Leverage/Inverse ETFs, as they mostly focus on VaR, risk-return relationship, investment performance, and market characteristics (e.g., Scatizzi, 2009; Sullivan, 2009; Militaru & Dzekounoff, 2010; Giese, 2010; Barnhorst *et al*., 2011; Zigler, 2013; DiLelli *et al*., 2014). Our study takes up the increasing importance and necessity of liquidity risk evaluation, especially for newly introduced Leverage/Inverse ETFs, for which a complete LaVaR measure is indispensable. Thus, the major contribution of this paper is to incorporate the endogenous liquidity risk effect and re-estimate the exogenous LaVaR using the Hellinger distance measure.

This section discusses the concepts of traditional LaVaR and the Hellinger distance measure. Section 2 modifies the endogenous liquidity risk on exogenous LaVaR through two sequential empirical models, which are traditional LaVaR model and endogenous liquidity risk-adjusted LaVaR model, at different trade size

[9] We include different trade size percentages at 1%, 3%, and 6% to broaden the scope of consideration; 1% is based on Simonian's (2011) research; 6% of the maximum value is based on the actual trading percentage; and 3% is average market trading percentage.

percentages (d_H) of 1%, 3%, and 6% by the Hellinger distance measure. Sections 3 and 4 explain the results of empirical models and compare the consistency of practical failure rates and their corresponding theoretical failure rates based on the consistency of the ex-post loss and ex-ante VaR according to back-testing results. We finally suggest implications for ETF investment decision-making.

2 Data and Methodology

2.1 Data

For empirical data we use the Taiwan 50 Leverage/Inverse ETFs and China SSE 180 Leverage/Inverse ETFs listed on the TWSE from October 14, 2014 to December 31, 2015, drawn from the TWSE daily historical data download page.[10] The empirical datasets are return rate and bid-ask mean spread, respectively calculated by the daily closing price and the daily bid and ask prices as Eqs. (1) and (2). Return rate and bid-ask mean spread are the major empirical variables in our research.

For measuring the heterogeneous volatility of the return and mean spread ($\sigma^2_{r,t}$ and $\sigma^2_{s1,t}$), we adopt Bollerslev's (1986) GARCH models as Eqs. (3) and (4) and then include the $\sigma^2_{r,t}$ and $\sigma^2_{s1,t}$ estimated results into Eqs. (5) and (6) respectively to measure the traditional VaR and COL. We incorporate traditional VaR and COL together as Eq. (7) to calculate the exogenous liquidity risk effect and traditional LaVaR. We apply Eqs. (8) and (9) to sensitize the different percentages of trade size effect at 1%, 3%, and 6%, relative to the 0% assumed, and revalue μ_{s2} and $\sigma^2_{s2,t}$ by the Hellinger distance measure. Adopting the new μ_{s2} and $\sigma^2_{s2,t}$, we measure the new COL as Eq. (10) and recalculate the endogenous liquidity risk effect on traditional LaVaR as Eq. (11). The empirical models are Eqs. (1) to (11) in sections 2.2 and 2.3, i.e., two major models, including the LaVaR model and the sensitizing analysis endogenous liquidity risk model using the Hellinger distance measure.

2.2 Traditional LaVaR

[10] Taiwan 50 Leverage/Inverse ETFs were listed on October 24, 2014; China SSE 180 Leverage/Inverse ETFs were listed on November 11, 2014.

The traditional LaVaR model is a simple and practical model that simultaneously measures asset pricing of mean-variance and adjusts for exogenous liquidity risk. This model applies the return rate of ETFs to only measure traditional VaR assumed market risk. The bid-ask mean spread is a proxy variable for exogenous liquidity risk to adjust the traditional VaR, called the cost of liquidity (COL). As in previous studies, we assume the probability distributions of returns and spreads follow a normal distribution. They are $N(\mu_{r,t}, \sigma_{r,t})$ and $N(\mu_{s1,t}, \sigma_{s1,t})$. We respectively calculate the return rate ($\mu_{r,t}$) and mean spread ($\mu_{s1,t}$) by Eqs. (1) and (2).

$$\mu_{r,t} = \ln(\frac{P_t}{P_{t-1}}), \quad (1)$$

$$\mu_{s1,t} = \frac{P_{b,t} - P_{a,t}}{\frac{P_{b,t} + P_{a,t}}{2}}, \quad (2)$$

where Eq. (1) is the one-day holding horizon return; P_t and P_{t-1} are the daily closing price in periods t and t-1; Eq. (2) is the one-day holding horizon bid-ask mean spread; and $P_{b,t}$ and $P_{a,t}$ are bid price and ask price in period t.

Due to the heterogeneous volatility of the return and mean spread, we build upon Bollerslev's (1986) GARCH models to estimate $\sigma^2_{r,t}$ and $\sigma^2_{s1,t}$. The empirical models are Eqs. (3) and (4):

$$\sigma^2_{r,t} = \alpha + \beta\sigma^2_{r,t-1} + \gamma\varepsilon^2_{r,t-1}, \quad (3)$$

$$\sigma^2_{s1,t} = \pi + \theta\sigma^2_{s1,t-1} + \tau\varepsilon^2_{s1,t-1}, \quad (4)$$

where Eq. (3) is the one-day holding horizon volatility of return $\sigma^2_{r,t}$; $\sigma^2_{r,t-1}$ and $\varepsilon^2_{r,t-1}$ are respectively the daily closing volatility and residual in periods t and t-1; Eq. (4) is the one-day holding horizon volatility of mean spread, $\sigma^2_{s1,t}$; and $\sigma^2_{s1,t-1}$ and $\varepsilon^2_{s,t-1}$ are respectively the daily mean spread volatility and residual in periods t and t-1.

We set up the traditional VaR model based on Bangia *et al.*'s (1999, 2001) and Simonian's (2001) empirical models, which only consider market risk without considering exogenous liquidity or endogenous liquidity, assuming the cost of liquidity and the trade size percentage are both zero. The measure calculation is Eq. (5):

$$\mathrm{VaR} = P_t * (1 - \exp(-1.96 * \eta_{r,t} * \sigma_{r,t})), \quad (5)$$

$$\eta_{r,t} = 1.0 + \varphi * \ln(\frac{\kappa_{r,t}}{3}),$$

where Eq. (5) is the one-day holding horizon of traditional VaR; P_t is the daily closing price in period t; and $\sigma_{r,t}$ is Std. Dev. of the return derived by Eq. (3). For precisely estimating the risk, we use the correction factor parameter "$\eta_{r,t}$" to modify the bias due to the non-normal distribution (non-N.D.), e.g., "leptokurtic" or "fat-tailed", of the probability density functions (PDF) of returns. Moreover, "$\kappa_{r,t}$" and "φ" are respectively the kurtosis coefficient and one-tailed probability 1%. When parameter $\eta_{r,t}$ equal 1 and $\kappa_{r,t}$ equals 3, the PDF of returns is N.D. and no adjustment is needed; when parameter $\eta_{r,t}$ and $\kappa_{r,t}$ are respectively greater than 1 and 3, the PDF of returns deviates significantly from normality, and an adjustment is needed (see Bangia *et al*. (1999, 2001)).

We next integrate the exogenous liquidity effect and calculate the cost of liquidity. As adjusted by Bangia *et al*. (1999, 2001) and Simonian (2001), the one-day COL formula is:

$$\mathrm{COL} = \frac{P_t}{2} * (\mu_{s1,t} + a * \sigma_{s1,t}), \quad (6)$$

where P_t is the daily closing price in period t; $\mu_{s1,t}$ is the mean spread derived by Eq. (2); and $\sigma_{s1,t}$ is Std. Dev. of the mean spread derived by Eq. (4); and "a" is the scaling-adjusted parameter for modifying the bias due to non-N.D. effects. We assume this parameter equals 2, 3, or 4.5 based on Bangia *et al*.'s (1999, 2001) and Simonian's (2011) models.

We incorporate traditional VaR and COL together in the traditional LaVaR model. The model could involve exogenous liquidity risk and traditional VaR. Thus, the LaVaR formula is Eq. (7) (Bangia, 1999, 2001; Shen *et al*., 2002; Simonian, 2011; Si & Fan, 2012; Tsai & Li, 2015; Tsai & Wu, 2016):

$$\mathrm{LaVaR} = \mathrm{VaR} + \mathrm{COL}, \quad (7)$$
$$\mathrm{LaVaR} = P_t * \left[(1 - \exp\left(-1.96 * \eta_{r,t} * \sigma_{r,t}\right)) + 0.5 * (\mu_{s1,t} + a * \sigma_{s1,t})\right].$$

2.3 Sensitized Endogenous Liquidity by Hellinger Distance Measure

Following Bogachev (2007) and Simonian (2011),[11] we apply Hellinger distance measure characteristics to measure the difference between traditional LaVaR and endogenous-adjusted LaVaR. The Hellinger distance is one of a family of "f-divergences", which estimate the distance in probability measures. The probability of this measure is a percentage of trade size and always falls within a range of $[0, 1]$. The Hellinger distance $H(P|Q)$ is equivalent to the percentage of market trade size (d_H), which is between 0% and 100%. The one-day holding horizon of Hellinger distance is Eq. (8):

$$H(P|Q) = d_H = \sqrt{1 - \int \sqrt{f(x)g(x)}dx}, \tag{8}$$

where f(x) and g(x) are two probability measures of the state space Ω, which could be used to measure the difference between traditional LaVaR and endogenous-adjusted LaVaR at given trade size percentages.

Moreover, when the probabilistic allocation is assumed to be absolutely continuous with a σ-finite dominating measure and with normal probability distributions $N(\mu_{s1,t}, \sigma_{s1,t})$ and $N(\mu_{s2,t}, \sigma_{s2,t})$ (as described by Lagarias *et al.*, 1998), we can refine the one-day holding horizon of Hellinger distance (d_H) as Eq. (9):

$$d_H = 1 - \left(\frac{2\sigma_{s1,t}\,\sigma_{s2,t}}{\sigma_{s1,t}^2 + \sigma_{s2,t}^2}\exp\left(-\frac{1}{4}\frac{(\mu_{s1,t} - \mu_{s2,t})^2}{\sigma_{s1,t}^2 + \sigma_{s2,t}^2}\right)\right)^{0.5}, \tag{9}$$

where $\mu_{s1,t}$ and $\sigma_{s1,t}$ are the mean spread and Std. Dev. derived by Eq. (4), which are parameters of probability distributions under exogenous liquidity risk (i.e., traditional LaVaR); and $\mu_{s2,t}$ and $\sigma_{s2,t}$ are respectively the mean spread and Std. Dev. estimated by Eq. (9) solved by the Nelder-Mead simplex algorithm in the Mathematica 10.0 program, which are parameters for probability distributions incorporating endogenous liquidity risk by the Hellinger distance measure at market trade sizes (i.e., d_H is1 %, 3%, and 6%). We solve the parameters $\mu_{s2,t}$ and $\sigma_{s2,t}$ using the Nelder-Mead simplex algorithm in the Mathematica 10.0 program.

We finally plug $\mu_{s2,t}$ and $\sigma_{s2,t}$ back into Eq. (10) to recalculate COL and use the new COL and traditional VaR to recalculate new LaVaR by Eq. (11). Equations (10) and (11) are as follows (Simonian, 2001):

[11] Refer to footnote 7.

$$\mathrm{COL} = \frac{\mathrm{P_t}}{2} * (\mu_{s2,t} + a * \sigma_{s2,t}), \tag{10}$$

$$\mathrm{LaVaR} = \mathrm{VaR} + \mathrm{COL}\,,$$
$$\mathrm{LaVaR} = \mathrm{P_t} * \left[\left(1 - e^{-1.96 * \eta_{r,t} * \sigma_{r,t}} \right) + 0.5 * (\mu_{s2,t} + a * \sigma_{s2,t}) \right]. \tag{11}$$

This research thus empirically investigates the liquidity-adjusted LaVaR using the Hellinger distance measure by sensitizing endogenous liquidity risk with trade sizes at 1%, 3%, and 6%. To include trade size in the model, we treat the trade size percentage as a proxy variable to rectify LaVaR. While previous studies assume the percentage is only 1% like in Simonian (2011), the present study expands this assumption to 1%, 3%, and 6%, where 1% is based on Simonian's (2011) research, 6% is the maximum value based on the actual trading percentage, and 3% is average market trading percentage.[12]

2.4 Back-testing

Traditional VaR and LaVaR typically estimate a day-to-day loss at a specified left-hand critical value of the portfolio's potential loss distribution. The recommended back-testing guideline proposed by BCBS (1996) evaluates a 1% VaR model over a 12-month test period of 250 trading days. (Chen *et al*., 2012) Thus, we assume the one-tailed probability is at 1% and the confidence level is at 95% to test the consistency between the practical violation rates and theoretical proportion of failures. For recording and accumulating the day-to-day proportion of failures (POF), we refer to Kupiec's (1995) opinion, denote the random variable "n" as the number of times for the whole empirical period, and record the consistency between daily ex-post losses and their ex-ante VaR, i.e., the consistency between the practical violation rates and theoretical proportion of failures. When the accumulated number of failures (i.e., ex-post loss is higher than ex-ante VaR) in a given period is $\hat{\alpha}$, and the POF recorded is $\hat{p}$, the PDF of POF is a binomial distribution and expressed as following Eq. (12):

$$\mathrm{Prob}(\hat{p}, \hat{\alpha}) = \hat{p}^{\hat{\alpha}} (1 - \hat{p})^{n - \hat{\alpha}}. \tag{12}$$

[12]We add the trade size percentages at 1%, 3%, and 6% as d_H to broaden the scope of consideration. The absolute value of d_H is between 0% and 100% and assumed to be equivalent to the trade size percentage (Bogachev, 2007 and Simonian, 2011).

By the likelihood ratio (LR_{POF}) unconditional coverage (UC) test, we examine the hypothesis $\hat{p} = p_0$ that the practical violation rate is equal to the theoretical proportion of failures for an accurate VaR forecasting method. We express the LR test statistics as follows (see Kupiec's (1995) and Gerlach *et al.*'s (2016) POF UC test):

$$LR_{POF} = \chi^2 - 2\text{Log}[p_0^{\alpha_0}(1-p_0)^{n-\alpha_0}] + 2\text{Log}\left[\hat{p}^{\hat{\alpha}}(1-\hat{p})^{n-\hat{\alpha}}\right], \quad (13)$$

where Eq. (13) is the LR statistics to test the consistency between daily ex-post losses and ex-ante VaR; and $\hat{\alpha}$, α_0, $\hat{p}$, and p_0 are respectively the number of practical failures, number of theoretical failures, practical proportion of failures, and theoretical proportion of failures.

3 Empirical Results

3.1 Descriptive Statistics

We draw the empirical data of Taiwan 50 and China SSE 180 Leverage/Inverse ETFs from the TWSE Web Database, each respectively providing 289 and 272 daily data samples. We then calculate the return and mean spread by Eqs. (1) and (2) and show the descriptive statistics results, including average and Std. Dev. value of price, return, ask price, bid price, bid-ask spread, and mean spread in Table 4.

Table 4. Leverage/Inverse ETFs' Descriptive Statistics and Test Results

ETFs	Taiwan 50 ETF											
	Leverage						Inverse					
	Price	Return	Ask price	Bid price	Spread	Mean spread	Price	Return	Ask price	Bid ask	Spread	Mean spread
No.	289	289	289	289	289	289	289	289	89	289	289	289
Mean	20.598	-0.00081	20.405	20.814	0.183	0.002	19.317	0.000028	19.212	19.418	0.207	0.003
Std. Dev.	2.061	0.039	2.101	2.045	0.182	0.003	0.861	0.010	0.823	0.921	0.157	0.002
ETFs	China SSE 180 ETF											
	Leverage						Inverse					
	Price	Return	Ask price	Bid price	Spread	Mean spread	Price	Return	Ask price	Bid ask	Spread	Mean spread
No.	272	272	272	272	272	272	272	272	272	272	272	272
Mean	40.663	0.00358	41.641	39.740	1.901	0.047	12.677	-0.0023	11.485	11.536	0.304	0.006
Std. Dev.	10.712	0.098	10.943	10.493	1.262	0.031	1.809	0.025	1.771	1.780	0.211	0.004

Note: Taiwan 50 Leverage/Inverse ETFs and China SSE 180 Leverage/Inverse ETFs are two ETFs listed on TWSE. They are described in footnotes 2 and 3.

We now examine the Taiwan 50 Leverage/Inverse ETFs. The average and Std. Dev. measures of price, bid price, ask price, bid-ask spread, and mean spread of the Leverage ETF are larger than those of the Inverse ETF. Therefore, the return of the Leverage ETF is negative, while that of the Inverse ETF is positive. These findings indicate the Leverage ETF has a lower return and is associated with higher risk, while the Inverse ETF has a higher return and is associated with lower risk. Subsequently, we look at the China SSE 180 Leverage/Inverse ETFs. The average and Std. Dev. measures of price, return, ask price, bid price, bid-ask spread, and mean spread of the Leverage ETF are also higher than those of the Inverse ETF. The return of the Leverage ETF is positive, while that of the Inverse ETF is negative. This indicates the Leverage ETF has a higher return and is associated with higher risk, while the Inverse ETF has a lower return and is associated with lower risk.

Comparing Leverage/Inverse ETFs in different price trends, the China SSE 180 Leverage ETF has a higher return and higher risk than that of the Taiwan 50 Leverage ETF in bullish markets. Furthermore, the China SSE 180 Inverse ETF has lower return and higher risk level than the Taiwan 50 Inverse ETF in bearish markets.

3.2 Empirical Result on Traditional VaR

This research uses return data as a proxy variable to measure the market risk level and calculate traditional VaR. We first build a Bollerslev (1986) GARCH regression model (Eq. (3)) for the Taiwan 50 ETFs and the China SSE 180 ETFs. The empirical results of normal dist. test statistics (Jarque & Bera, 1987), unit root test statistics (Dickey & Fuller, 1979), and heteroscedastic test statistics (Engle, 1982) presented in Table 5 show that the return of all ETFs follow non-N.D., stationarity, and heteroscedasticity. Furthermore, the empirical GARCH results of quasi maximum likelihood estimation (QMLE) (Berndt *et al*., 1974) show that in the case of Taiwan 50 Leverage/Inverse ETFs and China SSE 180 Leverage/Inverse ETFs, the prior period volatility return $\sigma^2_{r,t-1}$ significantly influences the current volatility $\sigma^2_{r,t}$. We estimate coefficient β as 0.844, 0.868, 0.644, and 0.446, respectively, for the two ETFs, i.e., the Taiwan 50 Leverage/Inverse ETFs and the China SSE 180 Leverage/Inverse ETFs. The forecasting accuracy test results (e.g., mean square error, MSE; mean absolute error, MAE; mean absolute percentage error, MAPE; Theil coefficient) indicate the out-of-sample prediction is poor. We then plug $\sigma^2_{r,t}$ into Eq. (5) and calculate the traditional VaR for each trade date. To evaluate the accuracy of the VaR model, we apply Kupiec's (1995) POF test by counting the number of exceptions and the practical failure rates.[13] The POF test is based on a null hypothesis that the practical failure rates are consistent with theoretical failure at a 1% one-tailed probability and 95% confidence level. Table 5 lists the likelihood ratio test statistics adopted for hypothesis testing and the empirical results. The major findings are as follows.

(a) Taiwan 50 Leverage/Inverse ETFs

The average $\mu_{r,t}$, $\sigma_{r,t}$, and $\kappa_{r,t}$ of the Taiwan 50 Leverage ETF are -0.004, 0.019, and 5.497, respectively. We use $\mu_{r,t}$ and $\sigma_{r,t}$ to calculate the traditional VaR and modify the bias due to the non-N.D. by the correction factor parameter $\eta_{r,t}$. The parameter $\eta_{r,t}$ is equal to 1.006, the average value of traditional VaR is 0.623, and the price adjusted by traditional VaR is 19.946. We then use the POF test to compare the consistency with ex-post losses and ex-ante VaR. There is a total of 18 exceptions among 289 observations (i.e., the practical failure rate of the Leverage ETF is 6.228%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical results suggest that the practical failure rate is significantly inconsistent with the practical failure rate at the 95% confidence level.

The average $\mu_{r,t}$, $\sigma_{r,t}$, and $\eta_{r,t}$ of the Taiwan 50 Inverse ETF are 0.00003, 0.009, and 6.488, respectively. We use $\mu_{r,t}$ and $\sigma_{r,t}$ to calculate the traditional VaR and modify the bias due to the non-N.D. by correction factor parameter $\eta_{r,t}$. The measure method is the same as that used for the Leverage ETF. The parameter $\eta_{r,t}$ is 1.008, the average value of traditional VaR is 0.300, and the price adjusted by traditional VaR is 18.870. We then use the POF test to compare the consistency with ex-post losses and ex-ante VaR. There is a total of 16 exceptions among 289 observations (i.e., the practical failure rate of the Leverage ETF is 5.536%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3),

[13]When the ex-post loss is higher than ex-ante VaR in a given period, we accumulate the number of failures $\hat{\alpha}$ and record the POF. By the likelihood ratio (LR_{POF}) of unconditional coverage (UC) test, we examine the hypothesis H_0: $\hat{p}=p_0$ at 1% one-tailed probability and 95% confidence level (Kupiec, 1995; Chan *et al*., 2012; Gerlach *et al*., 2016).

the empirical results suggest that the practical failure rate is significantly inconsistent with the practical failure rate at the 95% confidence level.

(b) China SSE 180 Leverage/Inverse ETFs

The average $\mu_{r,t}$, $\sigma_{r,t}$, and $\kappa_{r,t}$ of the China SSE 180 Leverage ETF are 0.002, 0.049, and 4.978, respectively. We use $\mu_{r,t}$ and $\sigma_{r,t}$ to calculate the traditional VaR and adjust the bias due to the non-N.D. by the correction factor parameter $\eta_{r,t}$. The parameter $\eta_{r,t}$ is equal to 1.005, the average value of traditional VaR obtained is 1.472, and the price adjusted by traditional VaR is 40.419. We then use the POF test to compare the consistency with ex-post losses and ex-ante VaR. There is a total of 18 exceptions among 272 observations in all trade sizes (i.e., the practical failure rate of the Leverage ETF is 6.638%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical result indicates that the practical failure rate is significantly inconsistent with the practical failure rate at the 95% confidence level.

The average $\mu_{r,t}$, $\sigma_{r,t}$, and $\kappa_{r,t}$ of the China SSE 180 Inverse ETF are -0.002, 0.025, and 3.087, respectively. We use $\mu_{r,t}$ and $\sigma_{r,t}$ to calculate the traditional VaR and adjust the bias due to the non-N.D. by correction factor parameter $\eta_{r,t}$. The measure method is the same as that used for the Leverage ETF. The parameter $\eta_{r,t}$ is equal to 1.001, the average value of traditional VaR obtained is 0.450, and the price adjusted by traditional VaR is 12.103. We then use the POF test to compare the consistency with ex-post losses and ex-ante VaR. There is a total of 20 exceptions among 272 observations in all trade sizes (i.e., the practical failure rate of the Inverse ETF is 7.353%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical result indicates that the practical failure rates are significantly inconsistent with the practical failure rate at the 95% confidence level.

3.3 Empirical Result on Traditional LaVaR

This research uses the bid-ask spread as a proxy variable to measure the exogenous liquidity risk and calculate the traditional LaVaR. We first build a Bollerslev (1986) GARCH regression model for the Taiwan 50 and the China SSE 180 ETFs as in Eq. (4). The empirical results in Table 6 show that the bid-ask spread of all ETFs follow non-N.D., the trend is stationary, and the variance is heteroscedastic. Furthermore, the empirical GARCH results of quasi maximum likelihood estimation (QMLE) (Bernd *et al*., 1974) present that in the case of Taiwan 50 Leverage/Inverse ETFs and China SSE 180 Leverage/Inverse ETFs, the prior period volatility return $\sigma^2_{r,t-1}$ significantly influences the current volatility $\sigma^2_{r,t}$. The coefficient θ is estimated as 0.883, 0.683, 0.501, and 0.643, respectively, for the four ETFs, i.e., the Taiwan 50 Leverage/Inverse ETFs and the China SSE 180 Leverage/Inverse ETFs. The predictive accuracy test results (e.g., MSE, MAE, MAPE, Theil coefficient) indicate the out-of-sample prediction is poor.

Table 5. Empirical Results for Leverage/Inverse ETFs using the Traditional VaR Model

ETFs	Taiwan 50 ETF		China SSE 180 ETF	
	Leverage	Inverse	Leverage	Inverse
Raw data test:[a]				
Normal dist. test result	34.556*	48.669*	32.649*	12.489*
Unit root test result	-16.207*	-16.326*	-16.289*	-13.318*
Heteroscedasticity test result	4.129*	4.735*	4.196*	4.530*
GARCH estimated result:[b]				
α	0.00003	0.000004	0.0004	0.000169*
β	0.844*	0.868*	0.644*	0.446*
γ	0.095	0.093	0.195	0.365*
QMLE	545.357	950.240	251.832	630.411
AIC	-5.139	-6.553	-1.830	-4.613
GARCH forecasted accuracy result:[c]	0.019	0.010	0.049	0.025
MAE	0.014	0.007	0.035	0.018
MAPE	87.230	87.232	89.632	89.26
Theil coefficient	0.923	0.908	0.913	0.908
Traditional VaR result:[d]				
Average $\mu_{r,t}$	-0.004	0.00003	0.002	-0.002
Average $\sigma_{r,t}$	0.019	0.009	0.049	0.025
Average $\kappa_{r,t}$	5.479	6.488	4.978	3.087
Average $\eta_{r.t}$	1.006	1.008	1.005	1.001
Average VaR	0.623	0.300	1.472	0.450
Average price	19.946	18.870	40.419	12.103
Kupiec's back-testing: Failure rate	6.228%*	5.536%*	6.638%*	7.353%*

Note: * reject H_0 at α=0.05.

a. The hypothesis of the autoregressive model is H_0^1: The autoregressive model is "N.D."; H_0^2: The time series is non-stationary; and H_0^3: The autoregressive model is not heteroscedastic.
b. The hypothesis of the GARCH model is H_0: α=0, β=0, and γ=0.
c. MAE, MPE, PMSE, and Theil coefficient are four statistical data showing the forecasting accuracy of GARCH.
d. Traditional VaR is defined by eq. (5). The hypothesis of Kupiec's back-testing is H_0: $\hat{p}=p_0$.

We plug $\sigma_{s1,t}^2$ into Eq. (6) to calculate COL for each trade date by three different scaling-adjusted parameters (i.e., “a” is 2, 3, and 4.5), which modify the estimated bias due to the combined non-N.D. effects. Therefore, we sum up COL and traditional VaR as traditional LaVaR in Eq. (7). To evaluate the accuracy of the VaR model, we apply Kupiec’s (1995) POF and back-testing to compare the consistency of the practical and theoretical failure rates at a 1% one-tailed probability and 95% confidence level.[14] The likelihood ratio test statistics adopted for the hypothesis test and the empirical results are in Table 6. Compared with the results of traditional LaVaR, we find the following.

(a) Taiwan 50 Leverage/Inverse ETFs

The average $\mu_{s1,t}$ and $\sigma_{s1,t}$ of the Taiwan 50 Leverage ETF are respectively 0.010 and 0.019. We use $\mu_{s1,t}$ and $\sigma_{s1,t}$ to evaluate the changes of exogenous risk by assuming the scaling-adjusted parameter is at 2, 3, and 4.5, the result of COL being between 0.594 and 1.209. We incorporate traditional VaR and COL to calculate the traditional LaVaR at between 1.217 and 1.832. The new prices adjusted by traditional LaVaR are between 19.391 and 18.774. We use the POF test to compare the consistency with ex-post losses and ex-ante VaR. The number of exceptions is 8, 4, and 0, respectively, when the scaling-adjusted parameter is at 2, 3, and 4.5 (i.e., the practical failure rates of the Leverage ETF are respectively 2.768%, 1.381%, and 0%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical results indicate that the practical failure rates are significantly consistent with practical failure rates at the 95% confidence level assuming the percentage of trade sizes at 3% only.

The average $\mu_{s1,t}$ and $\sigma_{s1,t}$ of the Taiwan 50 Inverse ETF are respectively 0.011 and 0.012, and COL is between 0.336 and 0.467. We then incorporate traditional VaR and COL to find LaVaR between 0.636 and 0.767. The new prices adjusted by traditional LaVaR are between 18.677 and 18.386. We use the POF test to compare the consistency with ex-post losses and ex-ante VaR. The number of exceptions is 8, 4, and 0, respectively, when the scaling-adjusted parameter is at 2, 3, and 4.5 (i.e., the practical failure rates of the Leverage ETF are respectively 2.768%, 1.381%, and 0%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical results indicate that the practical failure rates are

[14] Refer to footnote 13.

significantly consistent with practical failure rates at the 95% confidence level assuming the percentage of trade sizes at 3% only.

(b) China SSE 180 Leverage/Inverse ETFs

The average $\mu_{s1,t}$ and $\sigma_{s1,t}$ of the China SSE 180 Leverage ETF are respectively 0.047 and 0.055. We use $\mu_{s1,t}$ and $\sigma_{s1,t}$ to evaluate the changes of exogenous risk by assuming the scaling-adjusted parameter is at 2, 3, and 4.5 and COL is between 1.161 and 1.938. We then incorporate traditional VaR and COL and calculated the traditional LaVaR to be between 2.633 and 3.410. The new prices adjusted by traditional LaVaR are between 39.267 and 38.064. We use the POF test to compare the consistency with ex-post losses and ex-ante VaR. The number of exceptions is 9, 4, and 0, respectively, when the scaling-adjusted parameter is at 2, 3, and 4.5 (i.e., the practical failure rates of the Leverage ETF are respectively 3.249%, 1.444%, and 0%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical results indicate that the practical failure rates are significantly consistent with practical failure rates at the 95% confidence level assuming the percentage of trade sizes at 3% only.

The average $\mu_{s1,t}$ and $\sigma_{s1,t}$ of the China SSE 180 Inverse ETF are respectively 0.028 and 0.030, and COL is between 0.463 and 0.489. We then incorporate traditional VaR and COL to find LaVaR between 0.913 and 0.939. The new prices adjusted by traditional LaVaR are between 11.778 and 11.702. We use the POF test to compare the consistency with ex-post losses and ex-ante VaR. The number of exceptions is 10, 4, and 0, respectively, when the scaling-adjusted parameter is at 2, 3, and 4.5 (i.e., the practical failure rates of the Leverage ETF are respectively 3.610%, 1.444%, and 0%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical results indicate that the practical failure rates are significantly consistent with practical failure rates at the 95% confidence level assuming the percentage of trade sizes at 3% only.

Table 6. Empirical Results for ETFs using the Traditional LaVaR Model

ETFs	Taiwan 50 ETF		China SSE 180 ETF	
	Leverage	Inverse	Leverage	Inverse
Raw data test: [a]				
Normal dist. test result	18.783*	97.716*	23.192*	13.823*
Unit root test result	-16.112*	-9.189*	-7.945	-11.501
Heteroscedasticity test result	4.380*	4.592*	4.471*	4.172*
GARCH estimated result:[a]				
π	0.000013	0.000052	0.001185	0.000913
θ	0.883*	0.683*	0.501*	0.643*
τ	0.041	0.303*	0.177*	0.275*
QMLE	847.193	891.081	412.845	581.074
AIC	-5.842	-6.146	-3.014	-4.251
GARCH forecasted accuracy result:[b] MSE	0.013	0.013	0.056	0.030
MAE	0.009	0.011	0.046	0.026
MAPE	85.813	88.721	86.913	87.111
Theil coefficient	0.961	0.968	0.969	0.970
Scaling-adjusted parameters a=2:[c]				
Average $\mu_{S1,t}$	0.010	0.011	0.047	0.028
Average $\sigma_{S1,t}$	0.019	0.012	0.055	0.030
Average COL	0.594	0.336	1.161	0.463
Average LaVaR	1.217	0.636	2.633	0.913
Average price	19.391	18.677	39.267	11.778
Failure rate	2.768%*	2.768%*	3.249%*	3.610%*
Scaling-adjusted parameters a=3:[c]				
Average $\mu_{S1,t}$	0.010	0.011	0.047	0.028
Average $\sigma_{S1,t}$	0.019	0.012	0.055	0.030
Average COL	0.841	0.453	1.271	0.475
Average LaVaR	1.464	0.753	2.743	0.925
Average price	19.144	18.651	38.789	11.695
Failure rate	1.381%	1.038%	1.444%	1.444%
Scaling-adjusted parameters a=4.5:[c]				
Average $\mu_{S1,t}$	0.010	0.011	0.047	0.028
Average $\sigma_{S1,t}$	0.019	0.012	0.055	0.030
Average COL	1.209	0.467	1.938	0.489
Average LaVaR	1.832	0.767	3.410	0.939
Average price	18.774	18.386	38.064	11.702
Kupiec's back-testing: Failure rate	0%*	0%*	0%*	0%*

Note: * reject H_0 at α=0.05.

a. and b. Please see the explanations in Table 5.

c. COL and LaVaR are defined as eqs. (6) and (7). The hypothesis of Kupiec's back-testing is H_0: $\hat{p}=p_0$.

3.4 Empirical Result of the Hellinger Distance Measure on LaVaR

As described in section 4.3, when the scaling-adjusted parameter of traditional LaVaR is set at 3, the Taiwan 50 Leverage/Leverage ETFs and China SSE 180 Leverage/Leverage ETFs significantly pass the back-testing hypothesis. Thus, we use the traditional LaVaR with a scaling-adjusted parameter of 3 as a model to sensitize for the endogenous liquidity risk effect. For measuring the endogenous liquidity risk, we use the Hellinger distance measure d_H as a series assuming different trade sizes. Because the percentages of the Leverage/Inverse ETFs in TWSE are between 1% and 6%, and the average size is 3% in 2014 and 2015, we consider d_H at 1%, 3%, and 6% as proxy variables to measure the incremental adjustment of endogenous liquidity risk on traditional LaVaR. By combining the probability measure of the Hellinger distance measure characteristics considered in Bogachev's (2007) and Simonian's (2011) research, we plug d_H at 1%, 3% or 6%, $\mu_{s1,t}$ and $\sigma_{s1,t}$ together into Eq. (9) to estimate $\mu_{s2,t}$ and $\sigma_{s2,t}$, which provide a sensitivity analysis for the endogenous liquidity risk.

Table 7 shows the empirical results obtained from the Nelder-Mead simplex algorithm in Mathematica 10.0 by Eq. (9). By plugging $\mu_{s2,t}$ and $\sigma\%_{s2,t}$ estimates into the COL calculation as in Eq. (10), we recalculate the new average COL for all ETFs; and incorporating COL and traditional VaR as Eq. (11), we also recalculate the new LaVaR. To evaluate the accuracy of the new LaVaR model, we apply Kupiec's (1995) POF test based on a null hypothesis that the practical failure rates are consistent with theoretical failure rates at a 1% one-tailed probability and 95% confidence level.[15] The likelihood ratio test statistics adopted for the hypothesis test and the empirical results are in Table 7. The research results are as follows.

(a) Taiwan 50 Leverage/Inverse ETFs

The average COL and LaVaR of the Taiwan 50 Leverage ETF increase with the total market trade size percentage (i.e., 1%, 3%, and 6%). By using the Hellinger distance measure calculated, the new $\mu_{s2,t}$ is between 0.011 and 0.016, and the new $\sigma_{s2,t}$ is between 0.020 and 0.025. Using $\mu_{s2,t}$ and $\sigma_{s2,t}$ to calculate COL, the new COL is between 0.844 and 0.869, and the new LaVaR is between 1.467 and 1.492.

[15]Refer to footnote 13.

The new prices adjusted by new LaVaR are between 19.130 and 19.001. We then use the POF test to compare the consistency with ex-post losses and ex-ante VaR. The number of exceptions is 4 among a total of 289 observations in all trade sizes (i.e., the practical failure rate of the Leverage ETF is 1.381%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical results indicate that the practical failure rates are consistent with practical failure rates at the 95% confidence level assuming the percentage of trade sizes at 1%, 3%, and 6%.

Using the Hellinger distance measure calculated, the new $\mu_{s2,t}$ of the Inverse ETF is between 0.012 and 0.016, and the new $\sigma_{s2,t}$ is between 0.013 and 0.016. The new COL is between 0.454 and 0.470, and the new LaVaR is between 0.754 and 0.771. The new price adjusted by the new LaVaR is between 18.627 and 18.486. We then use the POF test to compare the consistency with ex-post losses and ex-ante VaR. There are 3 exceptions among a total of 289 observations in all trade sizes (i.e., the practical failure rate of the Leverage ETF is 1.038%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical results indicate that the practical failure rates are consistent with practical failure rates at the 95% confidence level assuming the percentage of trade sizes at 1%, 3%, and 6%.

(b) China SSE 180 Leverage/Inverse ETFs

The average COL and LaVaR of the China SSE 180 Leverage ETF increase with the total market trade size percentage (i.e., 1%, 3%, and 6%). Using the Hellinger distance measure calculated, the new $\mu_{s2,t}$ is between 0.049 and 0.051, and the new volatility $\sigma_{s2,t}$ is between 0.056 and 0.059. By plugging the new mean and volatility values to recalculate COL, the new COL is between 1.284 and 1.302. The revaluated LaVaR is between 2.756 and 2.774. The new price adjusted by the new LaVaR is between 38.644 and 37.901. We then use the POF test to compare the consistency with ex-post losses and ex-ante VaR. There are 3 exceptions among a total of 272 observations in all trade sizes (i.e., the practical failure rate of the Leverage ETF is 1.444%). With a theoretical failure rate of 1%, the empirical results indicate that the practical failure rates (i.e., the theoretical failure number of exceptions is 3) are consistent with practical failure rates at the 95% confidence level assuming the percentage of trade sizes at 1%, 3%, and 6%.

Table 7. Empirical Results for ETFs Sensitized to Trade Size in LaVaR

ETFs		Taiwan 50 ETF		China SSE 180 ETF	
Percentage of trade size:[a, b]		Leverage	Inverse	Leverage	Inverse
0%	Average $\mu_{S2,t}$	0.010	0.011	0.047	0.028
	Average $\sigma_{S2,t}$	0.019	0.012	0.055	0.030
	Average COL	0.841	0.453	1.271	0.475
	Average LaVaR	1.464	0.753	2.743	0.925
	Average price	19.144	18.651	38.789	11.695
	Kupiec's back-testing: Failure rate	1.381%	1.038%	1.444%	1.444%
1%	Average $\mu_{S2,t}$	0.011	0.012	0.049	0.029
	Average $\sigma_{S2,t}$	0.020	0.013	0.056	0.031
	Average COL	0.844	0.454	1.284	0.479
	Average LaVaR	1.467	0.754	2.756	0.928
	Average price	19.130	18.627	38.644	11.391
	Kupiec's back-testing: Failure rate	1.381%	1.038%	1.444%	1.444%
3%	Average $\mu_{S2,t}$	0.014	0.014	0.050	0.030
	Average $\sigma_{S2,t}$	0.023	0.015	0.057	0.033
	Average COL	0.864	0.461	1.293	0.482
	Average LaVaR	1.487	0.761	2.756	0.932
	Average price	19.105	18.598	38.301	11.388
	Kupiec's back-testing: Failure rate	1.381%	1.038%	1.444%	1.444%
6%	Average $\mu_{S2,t}$	0.016	0.016	0.051	0.032
	Average $\sigma_{S2,t}$	0.025	0.016	0.059	0.034
	Average COL	0.869	0.470	1.302	0.485
	Average LaVaR	1.492	0.771	2.774	0.936
	Average price	19.001	18.486	37.901	11.374
	Kupiec's back-testing: Failure rate	1.381%	1.038%	1.444%	1.444%

Note: * reject H_0 at α=0.05.

a. d_H is 0% as in the Bangia *et al*. (1999, 2001) model; 1% as in the Simonian (2011) model; 6% and 3% are the maximum and average value based on the actual trading percentage. The new $\mu_{s2,t}$ and $\sigma_{s2,t}$ are estimated by Eq. (9), i.e., d_H.

b. New COL and LaVaR are defined by eqs. (10) and (11). The hypothesis of Kupiec's back-testing is H_0: $\hat{p}=p_0$.

The new $\mu_{s2,t}$ of the Inverse ETF is between 0.029 and 0.032, and the new $\sigma_{s2,t}$ is between 0.031 and 0.034. The new COL value is between 0.479 and 0.485, and the new LaVaR is between 0.928 and 0.936. The new prices adjusted by the new LaVaR are between 11.391and 11.374. We then use the POF test to compare the consistency with ex-post losses and ex-ante VaR. There are three exceptions among a total of 272 observations in all trade sizes (i.e., the practical failure rate of the Leverage ETF is 1.444%). With a theoretical failure rate of 1% (i.e., the theoretical failure number of exceptions is 3), the empirical results indicate that the practical failure rates are consistent with practical failure rates at the 95% confidence level assuming the percentage of trade sizes at 1%, 3%, and 6%.

A comparison of Leverage/Inverse ETFs in different price trends finds that the China SSE 180 Leverage and Inverse ETFs have higher LaVaR levels than the values of Taiwan 50 Leverage and Inverse ETFs in bullish and bearish markets respectively. Thus, China ETFs have higher risk levels than Taiwan ETFs. The major research findings are as follows. (a) In a bullish market, the traditional LaVaR of Taiwan 50 Leverage ETF is 1.464, and the Hellinger distance measure considers the percentage of trade size effects by the endogenous liquidity risk is between 1.467 and 1.492. The traditional LaVaR of China SSE 180 Leverage ETF is 2.743, and the Hellinger distance measure considers the percentage of trade size effects by the endogenous liquidity risk is between 2.756 and 2.774. (b) In a bearish market, the traditional LaVaR of Taiwan 50 Inverse ETF is 0.753, and the Hellinger distance measure considers the percentage of trade size effects by the endogenous liquidity risk is between 0.754 and 0.771. The traditional LaVaR of China SSE 180 Inverse ETF is 0.925, and the Hellinger distance measure considers the percentage of trade size effects by the endogenous liquidity risk is between 0.928 and 0.936.

4 Conclusions

This paper presents an empirical model that follows Bangia *et al*. (1999, 2001) in dividing liquidity risk into exogenous and endogenous types. However, Bangia *et al*. (1999, 2001) exclusively emphasize COL and exogenous liquidity risk calculations, without accounting for endogenous liquidity risk. We thus incorporate COL calculation and modify the exogenous liquidity risk for the traditional VaR (i.e., using traditional LaVaR as a base model). Using Simonian's (2001) empirical concept, we then incorporate the Hellinger distance measure to calculate the effect of percentage of trade size and modify the endogenous liquidity risk for the exogenous LaVaR. Simonian (2011) only sensitizes the liquidity effect when the trade size is at 1%, and thus we recalculate the endogenous liquidity effect by sensitizing for different trade sizes on the new mean $\mu_{s2,t}$ and volatility $\sigma_{s2,t}$. We subsequently revalue COL and plug it into the new LaVaR. We include additional trade size percentages to broaden the scope of consideration of this research. Based on the increasing importance and necessity for liquidity risk evaluation, especially for the newly introduced Leverage/Inverse ETFs, the complete LaVaR measure is indispensable. Thus, the major contribution of this paper is to incorporate the endogenous liquidity risk effect and re-estimate the exogenous LaVaR using the Hellinger distance measure.

By combining the probability measure of the Hellinger distance characteristics, applying sensitivity analysis to involve the endogenous liquidity risk, and adjusting the traditional LaVaR considered at the exogenous liquidity risk, we assume the Hellinger distance is a percentage of trade size at 0%, like the traditional LaVaR case in Bangis *et al*. (1999, 2001), at 1% as in Simonian's (2011) research, and at 3% and 6% as the average and maximum percentages of Taiwan ETFs' trading size. Because TWSE is promoting the policy of "Financial Import Substitution", that makes many financial products introduced, which could be offered investing in Chinese securities markets or overseas popularly. In the future, there will be other variously overseas Leverage/Inverse ETFs listed on the Taiwan stock market. Therefore, follow-up researchers could conduct analyses on various new ETFs, with the Hellinger distance measure at different percentages of trading size, so as to examine the endogenous liquidity risk effect on traditional LaVaR. By verifying the various ETFs, the LaVaR assessment model would broaden the scope of risk consideration.

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