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Revisiting Perverse Effects on Exchange Rate Pass-Through

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Abstract

The effects of a change in the exchange rate on product prices are investigated using a static international duopoly model without product differentiation. A general condition is derived for perverse exchange rate pass-through assuming decreasing marginal costs for firms in two trading countries. The result is clarified on the basis of a new diagram for determining equilibrium supplies in the two countries.

Key words: exchange rate pass-through; international duopoly; decreasing marginal cost

JEL classification: F1; L1

1. Introduction

The effects of a change in the exchange rate on prices have been analyzed for various static and dynamic models of international oligopoly since the appearance of the pioneering work of Dornbusch (1987) (see Goldberg and Knetter, 1997, for a comprehensive survey and Tivig, 1996, and Gross and Schmitt, 2000, for dynamic models). Krugman (1984) demonstrated that import protection is export promoting within an international duopoly with several markets if both duopolists' marginal costs are decreasing (see Okuguchi and Serizawa, 1996, for an alternative proof). Consider a homogeneous international duopoly in which both duopolists have decreasing marginal costs. If one country depreciates its currency against that of the other, its imports decrease initially; consequently, its own production increases, which leads to lower marginal costs and lower product prices. On the other hand, a decrease in exports of the other country is likely to lead to a decrease in its production and a rise in its product prices. Hence, the perverse exchange rate path-through is likely to occur. If exchange rate path-through is perverse, the balance of payments in a country whose currency is depreciated may deteriorate contrary to expectations.

In this paper we analyze the possibility of perverse effects of a change in the exchange rate on prices in an international Cournot duopoly without product

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differentiation. We make our analysis simpler by adopting a new approach to determine equilibrium supplies in the two trading countries. In Section 2 we find a general condition for the emergence of perverse exchange rate path-through. In Section 3 we illustrate our result on the basis of a diagram which makes it possible to determine equilibrium supplies in the two countries.

2. Mathematical Analysis

Consider two countries, country 1 and country 2, assumed to produce identical goods which are marketed to both countries. In this paper we assume that there is only one firm in each country. Let x_{ij} be the amount of product made in country *i* and marketed to country *j*, $i \neq j$, $i, j = 1,2$. By definition, the total amount X_i marketed to country *i* is

$$X_i \equiv x_{1i} + x_{2i}, i = 1,2.$$

The firm's total profit in country 1 in terms of its own currency is

$$\pi_1 \equiv p_1(X_1)x_{11} + ep_2(X_2)x_{12} - C_1(x_{11} + x_{12}), \tag{1}$$

where p_1 (with $p_1' < 0$) is the product price in country 1 in terms of its own currency, p_2 (with $p_2' < 0$) is the product price in country 2 in terms of its own currency, *e* is the exchange rate (the value of country 2's currency in terms of country 1's currency), and C_1 is the cost function for the firm in country 1. The firm's total profit in country 2 (π_2) in terms of country 1's currency is defined as

$$\pi_2 \equiv p_1(X_1)x_{21} + ep_2(X_2)x_{22} - eC_2(x_{21} + x_{22}), \tag{2}$$

where C_2 is the cost function for the firm in country 2 in terms of country 2's currency.

In the following analysis we assume continuous differentiability of the relevant functions up to the necessary orders. Assume that the firms in each country behave as Cournot duopolists. Then the first order conditions for profit maximization for country 1's firm are given by

$$\frac{\partial \pi_1}{\partial x_{11}} = \left[p_1(X_1) + x_{11}p_1'(X_1)\right] - C_1'(x_{11} + x_{12}) = 0, \tag{3.1}$$

$$\frac{\partial \pi_1}{\partial x_{12}} = \left[e(p_2(X_2) + x_{12}p_2'(X_2))\right] - C_1'(x_{11} + x_{12}) = 0, \tag{3.2}$$

where we have assumed away a corner maximum. Similarly, we have as the first order conditions for country 2's firm

$$\frac{\partial \pi_2}{\partial x_{21}} = p_1(X_1) + x_{21}p_1'(X_1) - eC_2'(x_{21} + x_{22}) = 0, \tag{4.1}$$

$$\frac{1}{e}\frac{\partial \pi_2}{\partial x_{22}} = (p_2(X_2) + x_{22}p_2'(X_2)) - C_2'(x_{21} + x_{22}) = 0. \tag{4.2}$$

Now we introduce the following fundamental assumptions:

$$p_j'(X_j) + x_{ij}p_j''(X_j) < 0, \quad i, j = 1, 2\ , \tag{A.1}$$

$$max\{p_1',\ ep_2'\} < C_1'', \tag{A.2}$$

$$max\{\frac{1}{e}p_1',\ p_2'\} < C_2''. \tag{A.3}$$

We note that the second order condition is satisfied for country 1's firm under (A.1), (A.2), and (A.4) given below. Similarly, the second order condition holds for country 2's firm under (A.1), (A.2), and (A.5) defined below. If $p_1' > ep_2'$, (A.2) becomes

$$p_1' < C_1''. \tag{A.2′}$$

If on the other hand $p_1' < ep_2'$, (A.3) is rewritten as

$$p_2' < C_2''. \tag{A.3′}$$

The assumptions (A.1) and (A.2′) or (A.3′) have been widely used in analyzing the existence and stability of the Cournot oligopoly equilibrium in a closed economy (Hahn, 1962; Okuguchi, 1976; Okuguchi and Szidarovszky, 1999). We observe that (3.1) and (3.2) are implicit functions of four variables, x_{11}, x_{12}, X_1, X_2, and the exchange rate *e*. Considering X_1, X_2, and *e* as parameters at this stage and applying the implicit function theorem, we can solve these functions with respect to x_{11} and x_{12} as

$$x_{11} \equiv \varphi_{11}(X_1, X_2, e), \tag{5.1}$$

$$x_{12} \equiv \varphi_{12}(X_1, X_2, e), \tag{5.2}$$

where

$$\varphi_{11}^1 \equiv \frac{\partial \varphi_{11}}{\partial X_1} = \frac{(p_1' + x_{11}p_1'')(ep_2' - C_1'')}{\Delta_1} < 0, \tag{6.1}$$

$$\varphi_{11}^2 \equiv \frac{\partial \varphi_{11}}{\partial X_2} = \frac{eC_1''(p_2' + x_{12}p_2'')}{\Delta_1} \gtrless 0 \Leftrightarrow C_1'' \gtrless 0, \tag{6.2}$$

$$\varphi_{11}^3 \equiv \frac{\partial \varphi_{11}}{\partial e} = \frac{C_1''(p_2 + x_{12}p_2')}{\Delta_1} \lessgtr 0 \Leftrightarrow C_1'' \gtrless 0, \tag{6.3}$$

$$\varphi_{12}^1 \equiv \frac{\partial \varphi_{12}}{\partial X_1} = -\frac{C_1''(p_1' + x_{11} p_1'')}{\Delta_1} \gtrless 0 \Leftrightarrow C_1'' \gtrless 0, \tag{6.4}$$

$$\varphi_{12}^2 \equiv \frac{\partial \varphi_{12}}{\partial X_2} = -\frac{e(p_1' - C_1'')(p_2' + x_{12} p_2'')}{\Delta_1} < 0, \tag{6.5}$$

$$\varphi_{12}^3 \equiv \frac{\partial \varphi_{12}}{\partial e} = -\frac{(p_1' - C_1'')(p_2 + x_{12} p_2')}{\Delta_1} > 0, \tag{6.6}$$

and Δ_1, assumed to be positive, is defined as

$$\Delta_1 \equiv \begin{vmatrix} p_1' - C_1'' & -C_1'' \\ -C_1'' & ep_2' - C_1'' \end{vmatrix} > 0. \tag{A.4}$$

Note that as X_1 and X_2 are the sum of x_{11} and x_{21} and of x_{12} and x_{22}, respectively, neither (5.1) nor (5.2) is a reaction function. They are functions introduced to simplify the following mathematical analysis. Similarly, solving (4.1) and (4.2) with respect to x_{21} and x_{22} we get

$$x_{21} \equiv \varphi_{21}(X_1, X_2, e), \tag{7.1}$$

$$x_{22} \equiv \varphi_{22}(X_1, X_2, e), \tag{7.2}$$

where

$$\varphi_{21}^1 \equiv \frac{\partial \varphi_{21}}{\partial X_1} = -\frac{(p_1' + x_{21} p_1'')(p_2' - C_2'')}{\Delta_2} < 0, \tag{8.1}$$

$$\varphi_{21}^2 \equiv \frac{\partial \varphi_{21}}{\partial X_2} = -\frac{eC_2''(p_2' + x_{12} p_2'')}{\Delta_2} \gtrless 0 \Leftrightarrow C_2'' \gtrless 0, \tag{8.2}$$

$$\varphi_{21}^3 \equiv \frac{\partial \varphi_{21}}{\partial e} = \frac{C_2'(p_2' - C_2'')}{\Delta_2} < 0, \tag{8.3}$$

$$\varphi_{22}^1 \equiv \frac{\partial \varphi_{22}}{\partial X_1} = -\frac{C_2''(p_1' + x_{21} p_1'')}{\Delta_2} \gtrless 0 \Leftrightarrow C_2'' \gtrless 0, \tag{8.4}$$

$$\varphi_{22}^2 \equiv \frac{\partial \varphi_{22}}{\partial X_2} = -\frac{(p_1' - eC_2'')(p_2' + x_{12} p_2'')}{\Delta_2} < 0, \tag{8.5}$$

$$\varphi_{22}^3 \equiv \frac{\partial \varphi_{22}}{\partial e} = \frac{C_2' C_2''}{\Delta_2} \gtrless 0 \Leftrightarrow C_2'' \gtrless 0, \tag{8.6}$$

and Δ_2, assumed to be positive, is defined as

$$\Delta_2 \equiv \begin{vmatrix} p_1' - eC'' & -eC_2'' \\ -C_2'' & p_2' - C_2'' \end{vmatrix} > 0. \tag{A.5}$$

Given the exchange rate, the Cournot equilibrium values for X_1 and X_2 are identical to the solution of the following system of equations:

$$X_1 = \varphi_{11}(X_1, X_2, e) + \varphi_{21}(X_1, X_2, e), \tag{9.1}$$

$$X_2 = \varphi_{12}(X_1, X_2, e) + \varphi_{22}(X_1, X_2, e). \tag{9.2}$$

The solution is clearly a function of the exchange rate *e*. Suppose Δ, assumed to be positive, is defined as

$$\Delta \equiv \begin{vmatrix} 1-(\varphi_{11}^1+\varphi_{21}^1) & -(\varphi_{11}^2+\varphi_{21}^2) \\ -(\varphi_{12}^1+\varphi_{22}^1) & 1-(\varphi_{12}^2+\varphi_{22}^2) \end{vmatrix} > 0. \tag{A.6}$$

We can justify the validity of assuming Δ is positive as follows. Consider the following algorithm for computing the equilibrium values of X_1 and X_2:

$$\begin{cases} X_1(t+1,e) = \varphi_{11}(X_1(t), X_2(t), e) + \varphi_{21}(X_1(t), X_2(t), e) \\ \qquad\qquad \equiv \varphi_1(X_1(t), X_2(t), e) & (10.1) \\ X_2(t+1,e) = \varphi_{12}(X_1(t), X_2(t), e) + \varphi_{22}(X_1(t), X_2(t), e) \\ \qquad\qquad \equiv \varphi_2(X_1(t), X_2(t), e), & (10.2) \end{cases}$$

where *t* denotes time. Define $\varphi_i^j \equiv \partial\varphi_i / \partial X_j$, $i, j = 1,2$. If $\|x\|$ is the maximum norm for a vector *x*, the above dynamic system can be shown to be contractive, i.e.,

$$\|X(t+1,e) - X(t,e)\| < \|X(t,e) - X(t-1,e)\|,$$

and therefore

$$\lim_{t\to\infty} X(t,e) \to X^*(e)$$

if

$$|\varphi_1^1| + |\varphi_1^2| = |\varphi_{11}^1 + \varphi_{21}^1| + |\varphi_{11}^2 + \varphi_{21}^2| < 1 \tag{11.1}$$

and

$$|\varphi_2^1| + |\varphi_2^2| = |\varphi_{12}^1 + \varphi_{22}^1| + |\varphi_{12}^2 + \varphi_{22}^2| < 1 \tag{11.2}$$

are simultaneously satisfied, where $X^*(e)$ is the unique solution of (9.1) and (9.2) for a given *e*. Under our set of assumptions, (11.1) and (11.2) lead to

$$1-(\varphi_{11}^1+\varphi_{21}^1) > 1+\varphi_{11}^1+\varphi_{21}^1 > -(\varphi_{11}^2+\varphi_{21}^2)$$

and

$$1-(\varphi_{12}^2+\varphi_{22}^2)>1+\varphi_{12}^2+\varphi_{22}^2>-(\varphi_{12}^1+\varphi_{22}^1),$$

respectively. Hence, (A.6) follows.

Now we get the following comparative static expressions from (9.1) and (9.2):

$$\frac{dX_1}{de}=\frac{(\varphi_{11}^3+\varphi_{21}^3)(1-\varphi_{12}^2-\varphi_{22}^2)+(\varphi_{12}^3+\varphi_{22}^3)(\varphi_{11}^2+\varphi_{21}^2)}{\Delta}, \tag{12.1}$$

$$\frac{dX_2}{de}=\frac{(\varphi_{12}^3+\varphi_{22}^3)(1-\varphi_{11}^1-\varphi_{21}^1)+(\varphi_{12}^1+\varphi_{22}^1)(\varphi_{11}^3+\varphi_{21}^3)}{\Delta}. \tag{12.2}$$

The signs of these two expressions are, in general, indeterminate. However, if $C_1''=C_2''=0$ (i.e., there are constant marginal costs for the two firms), we have

$$\varphi_{11}^3+\varphi_{21}^3<0,\ 1-\varphi_{12}^2-\varphi_{22}^2>0,\ \varphi_{12}^3+\varphi_{22}^3>0,\ \varphi_{11}^2+\varphi_{21}^2=0,$$
$$\varphi_{12}^3+\varphi_{22}^3>0,\ 1-\varphi_{11}^1-\varphi_{21}^1>0,\ \varphi_{12}^1+\varphi_{22}^1=0,\ \varphi_{11}^3+\varphi_{21}^3<0.$$

Hence,

$$\frac{dX_1}{de}<0 \quad \text{and} \quad \frac{dX_2}{de}>0,$$

leading to

$$\frac{dp_1}{de}>0 \quad \text{and} \quad \frac{dp_2}{de}<0,$$

and the exchange rate pass-through is normal.

We now proceed to derive a set of sufficient conditions which leads to the perverse exchange rate pass-through. For this purpose we assume that the two firms' marginal costs are decreasing:

$$C_1''<0 \quad \text{and} \quad C_2''<0. \tag{A.7}$$

First, consider the numerator of the RHS of (12.1). Since under (A.7),

$$1-\varphi_{12}^2-\varphi_{22}^2>0 \quad \text{and} \quad \varphi_{11}^2+\varphi_{21}^2<0,$$

this expression is positive if

$$\varphi_{11}^3+\varphi_{21}^3=-\frac{C_1''(p_2+x_{12}p_2')}{\Delta_1}+\frac{C_2'(p_2'-C_2'')}{\Delta_2}>0 \tag{A.8}$$

and

$$\varphi_{12}^{3} + \varphi_{22}^{3} = -\frac{(p_1' - C_1'')(p_2 + x_{12} p_2')}{\Delta_1} + \frac{C_2' C_2''}{\Delta_2} < 0 \tag{A.9}$$

are simultaneously satisfied. Next, consider the numerator of the RHS of (12.2). Since under (A.7),

$$1 - \varphi_{12}^{2} - \varphi_{22}^{2} > 0 \quad \text{and} \quad \varphi_{12}^{1} + \varphi_{22}^{1} < 0$$

hold, the numerator is certain to be negative under (A.8) and (A.9). Hence, we have established the following theorem.

Theorem 1: If assumptions (A.1)-(A.7) hold, then the exchange rate pass-through is perverse under additional assumptions (A.8) and (A.9).

We give an intuitive explanation of this result as follows. Suppose e increases. The immediate effects of this is an increase in x_{12} and a decrease in x_{21}. If $C_1'' < 0$, an increase in x_{12} causes a decrease in C_1', which in turn is likely to lead to an increase in x_{11}. On the other hand, a decrease in x_{21} caused by an increase in e leads to an increase in C_2' in light of $C_2'' < 0$. Hence, x_{22} is likely to decrease. The ultimate effects of an increase in e on $X_1 \equiv x_{11} + x_{21}$ and $X_2 \equiv x_{12} + x_{22}$ are ambiguous. However, if (A.8) holds, X_1 increases and p_1 decreases. Moreover, if (A.9) holds, X_2 decreases and p_2 increases. Hence the perverse exchange rate path-through emerges.

To further clarify the economic implications of (A.8) and (A.9), rewrite them as

$$\frac{(p_1' - C_1'')(p_2 + x_{12} p_2')}{C_2' C_2''} < \frac{\Delta_1}{\Delta_2} < \frac{C_1''(p_2 + x_{12} p_2')}{C_2'(p_2' - C_2'')}. \tag{13}$$

We must have

$$C_1'' C_2'' > (p_1' - C_1'')(p_2' - C_2'') \tag{14}$$

for (13) to be non-void. Clearly, (14) holds if

$$p_1' > 2C_1'' \quad \text{and} \quad p_2' > 2C_2''. \tag{15}$$

Feenstra et al. (1996) have shown the relevance for exchange rate path-through of market size in the destination market in a model of international trade with product differentiation. We note, however, that (A.8), (A.9) and (13) say nothing about market size.

3. Diagrammatic Analysis

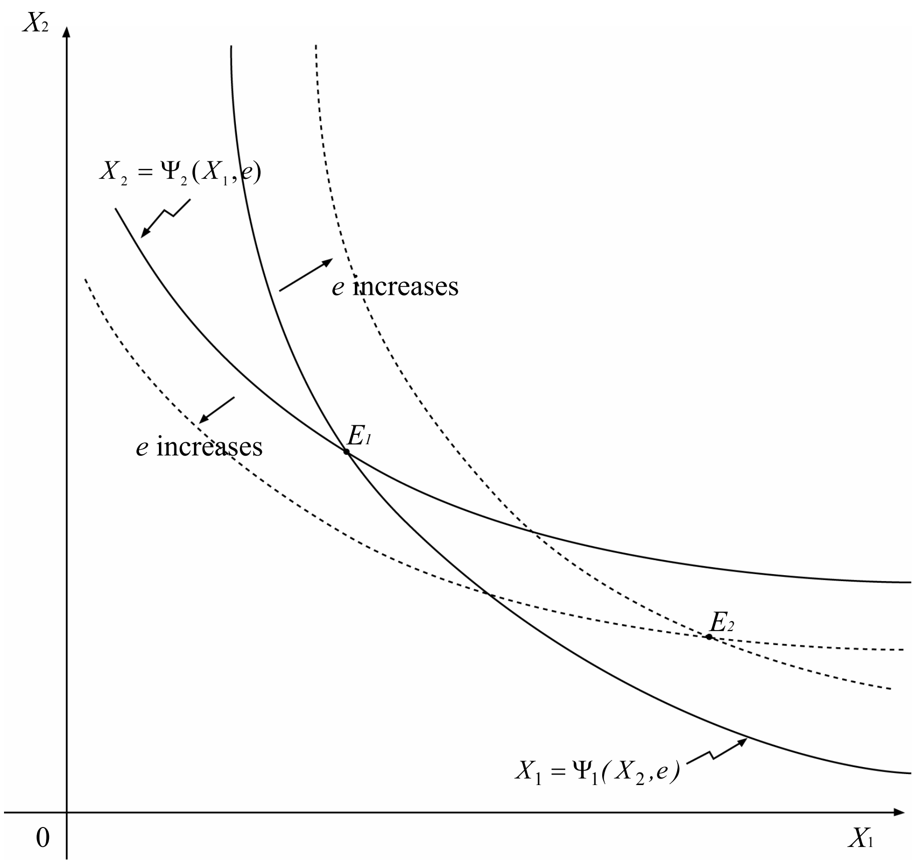
In this section we clarify our fundamental result on the perverse exchange rate pass-through under decreasing marginal costs by way of a diagram. Totally differentiating (9.1) and (9.2) and taking into account assumption (A.7), we obtain signs for the coefficients of dX_1, dX_2, and de as follows:

$$\underset{+}{(1-\varphi_{11}^1-\varphi_{21}^2)}dX_1=\underset{-}{(\varphi_{11}^2+\varphi_{21}^2)}dX_2+\underset{?}{(\varphi_{11}^3+\varphi_{21}^3)}de, \tag{16.1}$$

$$\underset{+}{(1-\varphi_{12}^2-\varphi_{22}^2)}dX_2=\underset{-}{(\varphi_{12}^1+\varphi_{22}^1)}dX_1+\underset{?}{(\varphi_{12}^3+\varphi_{22}^3)}de, \tag{16.2}$$

where "?" indicates the indeterminacy of the sign.

Figure 1. Determination of Equilibrium and the Effect of a Change in *e*



Denote the solution of (9.1) with respect to X_1 by

$$X_1 \equiv \Psi_1(X_2, e), \tag{17.1}$$

and that of (9.2) with respect to X_2 by

$$X_2 \equiv \Psi_2(X_1, e). \tag{17.2}$$

If we plot X_1 along the horizontal axis and X_2 along the vertical, the curves for both (17.1) and (17.2) become downward-sloping, as shown in Figure 1. Furthermore, if (A.6) holds, the curve for (17.1) has a steeper slope than that for (17.2). Hence, given the exchange rate *e,* the two curves are shown as the solid ones in the figure. The equilibrium is given by the intersection E_1 of the two curves. Under (A.8), if *e* increases (depreciation of country 1's country), the curve for (17.1) shifts upward, and under (A.9), that for (17.2) shifts downward as shown by the two dotted curves. The new equilibrium becomes E_2, yielding a larger X_1 (a lower p_1) and a smaller X_2 (a higher p_2). This is nothing but the perverse exchange rate pass-through. We note in passing that if the inequality in (A.8) is reversed, the curve for (17.1) shifts downward with an increase in *e*. On the other hand, if the inequality in (A.9) is reversed, the curve for (17.2) shifts upward with an increase in *e*. Hence, in this case the exchange rate pass-through is normal.

4. Conclusions

In this paper we analyzed the effects of a change in the exchange rate on the product prices of two trading countries producing identical products and marketing products to both countries. In general, the effects are ambiguous. If the marginal costs in two countries are constant, the effects of a change in the exchange rate are normal in the sense that the price of the good in the country that depreciates its currency increases and the price in the other country decreases. However, if the marginal costs in the two countries are both decreasing, the price of the goods in the depreciating country decreases while that in the other country increases under our set of general assumptions (A.1)-(A.7) if the additional assumptions (A.8) and (A.9) are satisfied. Tivig (1996) and Gross and Schmitt (2000) analyzed exchange rate path-through using dynamic models. Tivid (1996), like Feenstra et al. (1996), observed the importance of market size in the two-period model. In this paper we analyzed exchange rate path-through using only a static trade model without product differentiation. The dynamic extension of our analysis here is a problem for a separate paper.

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