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Rationing as a Signal

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Abstract

Two consumers sequentially purchase at most one unit of some homogeneous good from a monopolist who knows the state of nature, either high or low. I characterize a rationing equilibrium at which the high-type monopolist produces only one unit and rations customers, whereas the low-type monopolist serves customers by producing two units.

Key words: rationing; quality; signals; sequential purchases

JEL classification: D45; L12; L15

1. Introduction

A firm often creates excess demand and rations demand by setting a non-market clearing price. Examples are abundant. Lots of youngsters had difficulty in obtaining Harry Potter and the Goblet of Fire, the fourth volume in the Harry Potter series, in the summer of 2000 because it was sold out as soon as it went on sale. The publishers, however, neither responded to that immediately by supplying more stocks nor raised the price before it was out of stock. It is well known that Nintendo induced excess demand purposely by strictly controlling the supply of game cartridges when it introduced the Super Mario Brothers into the market in 1989. K-Paul's, a restaurant in New Orleans, is famous for long queues of customers outside it.

Recently, there have been theoretical approaches to explaining the rationing phenomenon as a result of a firm's rational behavior. A short (not exhaustive) list of articles explaining equilibrium rationing includes Kanneman, Knetsch, and Thaler (1986), Boyer and Moreaux (1988), Becker (1991), DeGraba (1995), Denicolò and Garella (1999), Gilbert and Klemperer (2000), etc.

Among others, Becker (1991), in a seminal short paper along this line, assumes

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that a person's demand for a product may increase as there are more people who want to buy it, i.e., individual demand may depend on the market demand. Then, the market demand curve may have an upward slope if this effect dominates the price effect, which can generate excess demand. DeGraba (1995) asserts that a firm can induce consumers to buy at a higher price by producing less than the demand on purpose when the consumers' valuations are realized later. Consumers, who know that the supply is limited, fear that products may be out of stock soon, rushing to buy one in the first period, which is called buying frenzies, defined by DeGraba as a consumer's behavior of purchasing before he learns his valuation. Gilbert and Klemperer (2000) argue that a buyer who made specific investments cannot get a surplus if the price is determined at the market clearing level, so that the price must be lower than the market clearing level in order to give a buyer the incentive to invest.

In this paper I will provide an alternative explanation of equilibrium rationing. I assert that some information should be transmitted by rationing. In this respect, I consider a model in which a firm has superior information to customers and show that the firm may produce less than the number of customers and ration customers to signal high quality (or high cost). In this situation, rationing can induce a later customer to purchase. One can often observe this business strategy being adopted in luxury brands like Ferrari, Piaget, Patek Philippe, and Montblanc that sometimes place limited editions on the market. (See "Phil Patton: Special Team Cars are Looking for a Few Good Buyers," New York Times, May 19, 1999.) Michael Jordan's basketball shoes can be another example.

Closest to this work in its spirit is DeGraba (1995) in the sense that customers' purchasing decisions are sequential. In his model, however, the role of signaling is not present because the firm does not know customers' valuation either. Also, in his model, no customer will buy the product in the second period because there are buying frenzies due to rationing in the first period. Here, rationing does not induce buying frenzies in the first period but affects the decision of the second-period customer.

The organization of the paper is as follows. In Section 2, I set up the model. In Section 3, a rationing equilibrium is characterized. In Section 4, I discuss the role of key assumptions and the refinement of equilibria. Concluding remarks follow in Section 5.

2. The Model

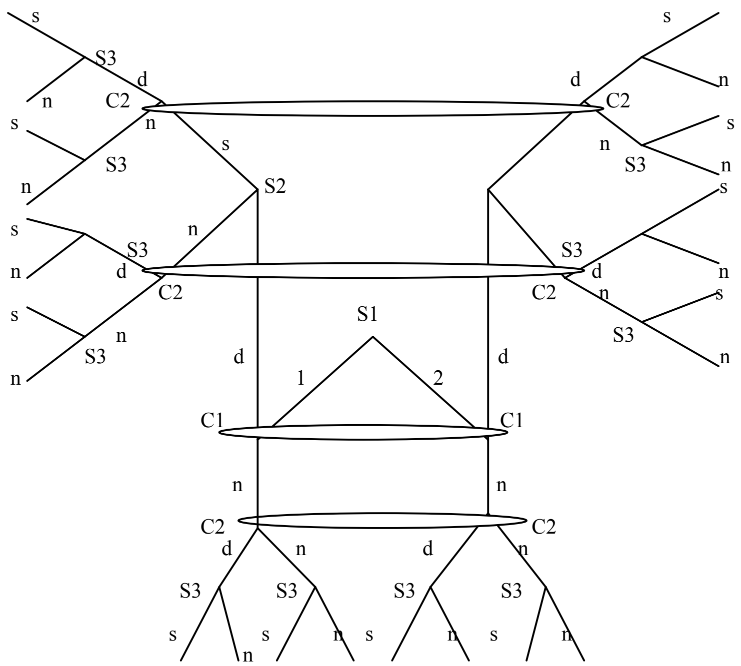
There are two consumers sequentially purchasing at most one unit of some homogeneous good from a monopolist who knows the state of nature: either high (*H*) or low (*L*). In either state, the monopolist must decide whether one or two units of output will be produced. Let high (low) cost in production be associated with high (low) value of satisfaction, specifically $c_H > c_L$ (> 0) and $v_H > v_L$. For simplicity, I assume that $v_L = 0$. Denote by λ taken from (0,1) the prior probability of having state *H*. Assume that λ, c_H, c_L, v_H, and v_L are all common knowledge, while only the monopolist knows the state of nature.

At the pre-trade stage, the output decision made by the monopolist is unknown to consumers yet the price chosen by the monopolist is observed. At the next stage, called the trade stage, two consumers sequentially purchase one unit from the monopolist at that price while random rationing is allowed.

We assume that the second customer may either observe the trading outcome of the previous customer (whether he attempted to purchase one and whether he managed to obtain one) with probability γ, or may not with probability $1-\gamma$, where γ may be interpreted as the proportion of consumers who can observe other consumers in the entire population. A game tree is depicted in Figure 1. The game starts from node denoted by *S1*. Here, I deliberately omit the Nature's move and present a game tree in (a) and (b), reflecting consumer 2's observability. Since the seller's information about consumer 2's observability is inessential, this generates no problem at all.

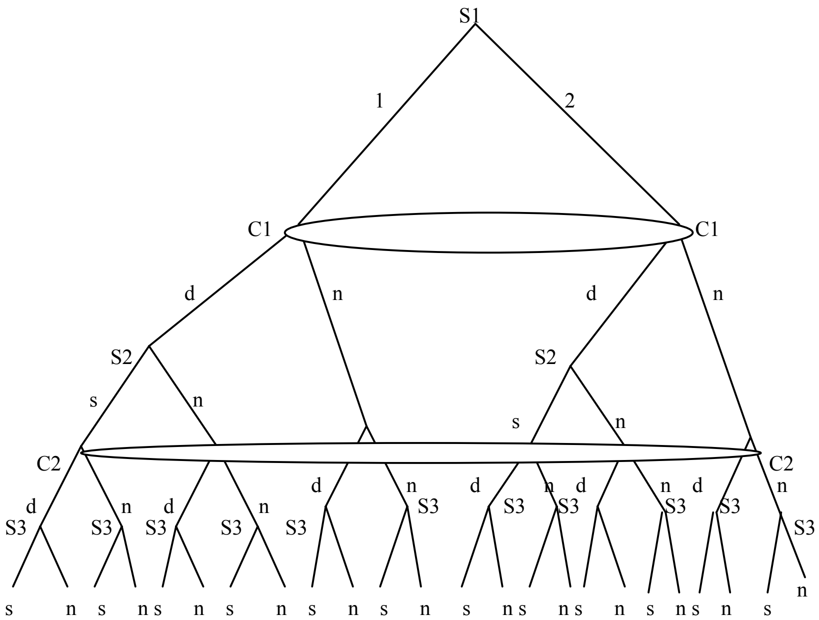
Notice that I distinguish production and sales. So, in the trade stage, production costs are already sunk. This feature turns out to be crucial to the analysis.

Figure 1a. Game Tree with Consumer 2 (Who Can Observe)



C1=consumer 1
C2=consumer 2
S1=1st move of the seller
S2=2nd move of the seller
S3=3rd move of the seller
d=demand
s=sell
n=not demand, not sell

Figure 1b. Game Tree with Consumer 2 (Who Cannot Observe)



3. Equilibrium Characterization

There may be multiple equilibria depending on various parameter values (rationing equilibria and non-rationing equilibria), but my interest will be mainly on the existence of the equilibrium where a monopolist rations customers. Hence, I will confine my attention to the equilibrium where both types of monopolist charge the same price p, where $p \in (v_L, v_H)$. Below, I will establish the result that there exists a rationing equilibrium in which the high-type monopolist produces only one unit and rations customers, whereas the low-type monopolist serves both customers by producing two units. For this purpose, I focus on a situation in which the following conditions are met.

Condition 1. $(1-\lambda)(v_L - p) + \lambda r(v_H - P) > 0$, where *r* is the probability that the high-type monopolist sells his product to customer 1.

Condition 2. $(1-\lambda)(v_L - p) + \lambda(1-r)(v_H - P) > 0$.

Condition 1 implies that customer 1 does demand one unit and Condition 2 implies that customer 2 who failed to observe the trading outcome of customer 1 also demands a unit.

3.1 Selling Decision

At each period, either type of monopolist can do no better than selling one unit because the production cost is sunk. Of course, he might be indifferent between selling and not selling if he can sell one unit at next period with certainty. In this case, he will randomize with probability r, which is possible only in the first period.

3.2 Consumer 2's Buying Decision

Consumer 2's decision must be contingent on the history, which consists of three possible events: observing the monopolist not selling to consumer 1 who demands one, observing the monopolist selling to him, or failing to observe at all.

On the first contingency, consumer 2 must believe that $v = v_H$, since $r < 1$ from Condition 2. So, he will demand one by Condition 1. On the second contingency, consumer 2 will not demand one. The high-type monopolist who sold one in the first period will not sell in the second period, while the consumer gets $v_L - p < 0$ if $v = v_L$. On the third contingency, the expected payoff of consumer 2 when he attempts to buy one is $(1-\lambda)(v_L - p) + \lambda(1-r)(v_H - p)$, so that he will also demand one, which is implied by Condition 2.

3.3 First Period Decision

As argued above, r cannot be one. That is, a high-type monopolist must ration his product to the customer in the first period.

3.4 Pre-Trade Stage

If a high-type monopolist produces two units, his expected profit will be $p + (1-\gamma)p - 2c_H$, which cannot exceed the expected profit when he produces only one unit, $p - c_H$, if $(1-\gamma)p - c_H \le 0$, i.e., $p \le (CH/1-\gamma)$. On the other hand, a low-type monopolist cannot benefit by producing one unit only if $p + (1-\gamma)p - 2c_L \ge p - c_L$, i.e., $p \ge (CH/1-\gamma)$.

Finally, consider the monopolist's pricing decision. It must be checked that neither type of monopolist has an incentive to deviate from the pooling equilibrium price p. This is trivial if one assigns the whole probability mass on the low type after observing any price other than p (reasonableness of this belief will be discussed in Section 4). Then, the maximum deviant price that a monopolist could charge would be $v_L (= 0)$. Since $p > v_L$, neither type would profitably deviate from p.

To summarize, I have the following theorem.

Theorem 1. There exists an equilibrium in which *H*-type monopolist produces one unit and *L*-type produces two units. This equilibrium is supported by $p \in \left[C_L/1-\gamma, C_H/1-\gamma\right] \cap \left[c_H, \infty\right]$ and $r \in (\rho, 1-\rho)$, where

$$\rho = -\frac{1-\lambda}{\lambda}\frac{v_L - p}{v_H - p}.$$

Proof: What remains to be shown is to check $r \in (\rho, 1-\rho)$, which is straightforward from Conditions 1 and 2.

This theorem says that the high type can signal high quality to the second customer by rationing the first customer i.e., selling him randomly. The crucial feature that enables a high type to be separated from a low type is a cost difference. It may be too costly for a high type to produce two units and not to sell them all, whereas it is less costly for a low type.

Also, Condition 1 warrants that consumer 1 will demand one unit.

4. Discussion

Imperfect Observability

In this model, the coexistence of consumers who can and cannot observe other consumers is essential to support the equilibrium I characterized. It is optimal for the high type to signal quality by producing less due to the existence of the former consumers, while it is optimal for the low type to serve both consumers by producing two units due to the existence of the latter consumers. However, as γ is larger, the range of equilibrium pooling prices is smaller, and, in particular, if $\gamma = 1$, this equilibrium range is degenerated. Intuitively, as γ is smaller, consumer 2 is less likely to observe consumer 1 and so buy one in the second period. If consumer 2 can perfectly observe consumer 1, he will know that the product he can buy in the second period is only L, so that he will not buy one in the second period, which implies that it is in the best interest of the low type to produce only one.

Uncertainty of the type

If λ is close to 0, that is, consumers believe strongly that the product is of low quality, Conditions 1 and 2 will not hold and consumers will not attempt to buy one either period. So, in this case, neither type of monopolist will produce any amount of the product.

Separation of Production and Sales

If the monopolist can produce a good whenever a consumer demands one after the price is set, the profit of high type can be increased by producing one unit and a second one if and only if it is demanded, rather than by producing only one and rationing, provided that Condition 1 holds.

Out-of-Equilibrium Belief

Rationing equilibria characterized in this paper rely on the most pessimistic out-of-equilibrium belief that a deviant price implies L. It is worthwhile to examine whether the out-of-equilibrium belief is reasonable.

Strictly speaking, straightforward application of the concepts of standard refinements to this model is not possible, because they are defined for the case that all the actions taken by the informed party are observable. Difficulty occurs in this model, since consumers can at most infer the output decision of the monopolist after the first consumer formed his posterior belief based on his observation of the price. However, if one extends the idea of standard refinements in a reasonable way, it can be asserted that it is hard to regard the out-of-equilibrium I assigned as unreasonable.

For example, the Intuitive Criterion proposed by Cho and Kreps (1987) cannot eliminate the equilibrium I characterized, in that any price other than p is equilibrium dominated by neither L or H. To see this, consider a deviation to $p+\varepsilon$ for a small $\varepsilon > 0$. A low type can get higher profits, $2(p+\varepsilon)-2c_L$, by such a deviation than equilibrium profits, $p+(1-\gamma)p-2c_L$, if consumers believe that the monopolist is H after observing $p+\varepsilon$. In this case, the posterior belief that the monopolist is H does not automatically imply that it produced only one unit because the information set corresponding to the observation of $p+\varepsilon$ can be reached along any off-the-equilibrium path. (More specifically, this information set can be reached even when H-type monopolist produced two units.) Thus, both consumers will demand one because they have nothing to lose by doing so. Similarly, a high type can increase its profit slightly by deviating to $p+\varepsilon$ if the deviation makes consumers believe that the monopolist is H. Similar arguments can be applied to deviations to $p-\varepsilon$.

Another refinement that is worth considering is the concept of universal divinity by Banks and Sobel (1987), which is more refined than the Intuitive Criterion. Suppose consumers react to the price $p+\varepsilon$ by demanding one unconditionally. Then, a low type gains more than a high type whose gain is only ε. Thus, universal divinity does not impose the belief that the monopolist who charges price $p+\varepsilon$ is H either.

It is trickier to discuss the reasonable belief of the equilibrium path after the first consumer failed to get one. In this case, one has two seemingly conflicting signals: rationing, which supports the belief of H, and the price $p+\varepsilon$, which supports the belief of L. However, if the second consumer observes the price $p+\varepsilon$ and knows that the monopolist produced only one unit, he can reasonably infer that the monopolist must be of high type, as long as $\varepsilon<(1-\gamma)p-c_L$, because the low type gets lower profits, $p+\varepsilon-c_L$, than equilibrium profits, $p+(1-\gamma)p-2c_L$, while the high type's profit is increased by ε.

This reasoning does not break my equilibrium, though, because the posterior belief formed in the first period that the monopolist is L discourages the first consumer from demanding one, thus inducing no rationing observed in equilibrium.

5. Conclusion

In this note, I present a model of equilibrium rationing and characterize an equilibrium where a high-type monopolist rations demand to signal the high quality (or high production cost) of his product. The result that only the high-type firm voluntarily induces excess demand in equilibrium may be worth empirically testing.

One clear extension of this research is to consider a model where duopolists compete with each other by adopting this strategy of inducing excess demand. More enriched models will be anticipated.

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